Department of Mathematical Sciences

Multinomial functional regression with application to lameness detection for horses

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useR! 2015, Ålborg

Background and aim for case study

Data collected by veterinarian **Maj Halling Thomsen** and her colleaugues at University of Copenhagen.

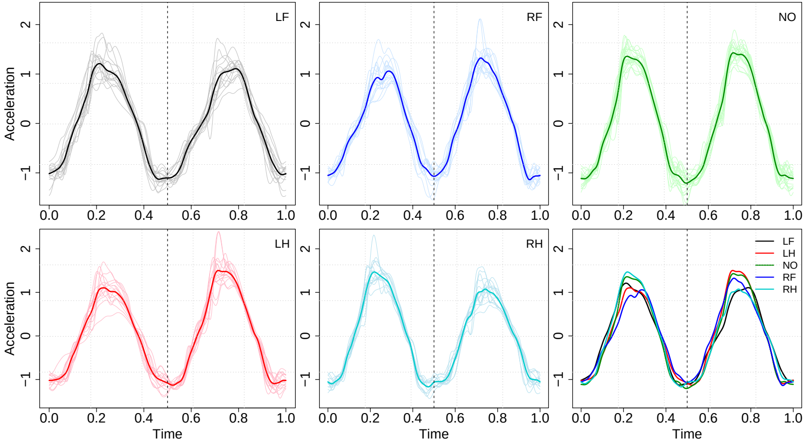
Problem: Difficult to detect lameness and identify the injured limb, especially for low-degree lameness

- Visual inspection and clinical examination is highly subjective
- Large variability in assessments from different veterinarians, and also between repeated assessments from the same vet

Aim: Develop an objective method for lameness detection

Detection of lameness: The movie

85 data signals (after preprocessing)



- Each curve: 'Average' vertical acceleration for one gait cycle
- First half: Stance on right-fore/left-hind diagonal

Problem and approach

Classification for functional data: Given a data signal, which group does it belong to?

Our approach:

- **Functional multinomial regression using wavelets and LASSO**
- Idea: wavelets offer precise approximations with sparse representations; use LASSO to induce sparseness
- Extension of the work by Zhao, Ogden, Reiss (2012)

Multinomial functional regression (MFR)

Data: Covariate functions $x_i(t)$, categorical outcomes $y_i \in \mathscr{M}$

Regression model

- Linear predictor for group m: $\eta_m(x) = \alpha_m + \int \beta_m(t)x(t)dt$
- Conditional probabilities:

$$p_m(x) = P(Y = m|X = x) = \frac{e^{\eta_m(x)}}{\sum_{l \in \mathscr{M}} e^{\eta_l(x)}}$$

- Unknowns: Intercepts α_m and coefficient functions $\beta_m(t)$

Classification of new curve, x_0: Compute $\hat{\eta}_m(x_0)$ and $\hat{p}_m(x_0)$ for all groups, and assign x_0 to group with largest probability

Estimation and computations

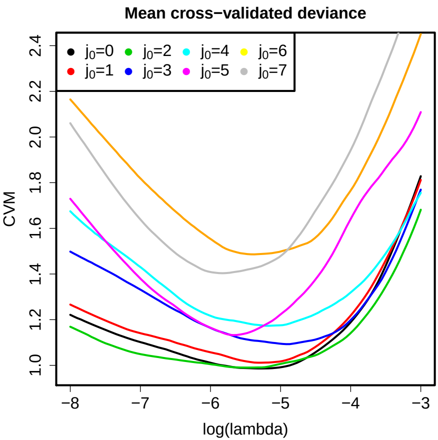
Estimation approach

- Expand functions x_i in a **wavelet basis**
- Use wavelet coefficients as covariates in ordinary multinomial regression with **LASSO penalization**
- Translate estimated regression coef's to coef. functions $\hat{\beta}_m$
- Tuning parameters are selected with **cross validation:** resolution level j_0 in wavelet basis and penalty parameter λ

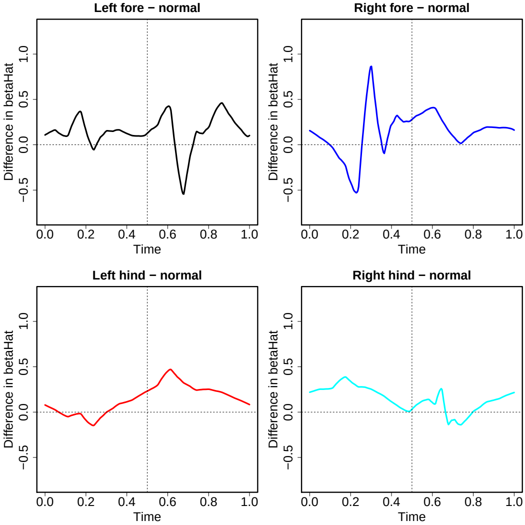
Computations

- `fda` package for preprocessing
- `wavethresh` package for wavelet computations
- `glmnet` package for penalized multinomial regression

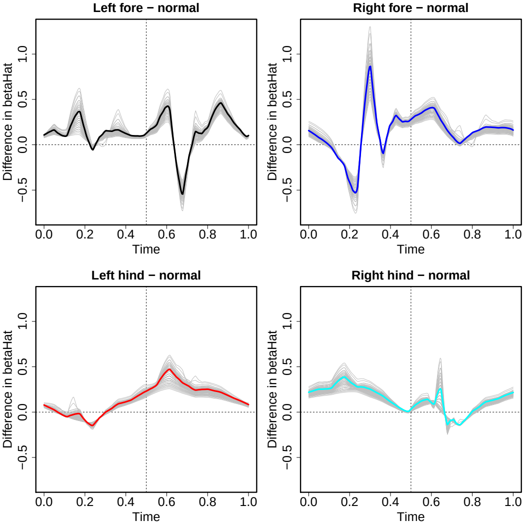
Cross validation results



Estimated coefficient functions: $\hat{\beta}_m - \hat{\beta}_{\text{no}}$



... and 100 bootstrap samples



Classification results

Leave-one-out classification:

	Predicted group				
True group	LF	LH	NO	RF	RH
LF	**13**	0	2	0	1
LH	1	**12**	2	1	0
NO	1	1	**19**	1	1
RF	0	2	1	**13**	0
RH	1	0	2	0	**11**

- 68 curves classified to correct group (80%); five more curves to correct diagonal; only one curve to wrong diagonal
- PC-LDA and method by Ferraty+Vieu (2003): $\approx$ 65% correct

Simulation study with similar data: 89% correct classifications

Discussion

- Estimated $\hat{\beta}_m$'s satisfy, to some extend, natural **symmetry restrictions.** Possible to build in those restrictions explicitly.
- MFR performs at least as well as the other methods we have tested — also in application from speech recognition
- Many possible **variations** which we didn't pursue
- Data come from only eight horses! Calls for **random effect of horse** — at least for inference about coefficient functions.

Thank you for your attention

Questions and comments are most welcome: `helle@math.ku.dk`

References

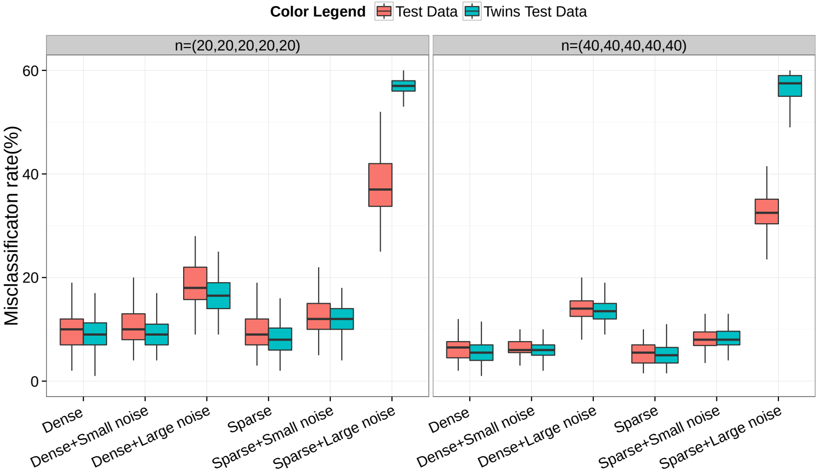
Mousavi, S. N., and Sørensen, H. (2015). Multinomial functional regression with wavelets and LASSO penalization. Submitted.

Zhao, Y., Ogden, R. T., and Reiss, P. T. (2012). Wavelet-based LASSO in functional linear regression. *Journal of Computational and Graphical Statistics*, 21:600–617.

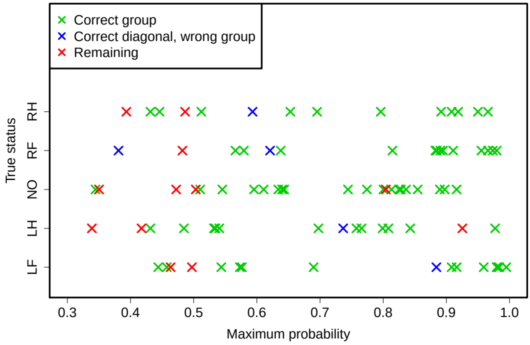
Friedman, J., Hastie, T., and Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of statistical software*, 33(1):1.

Ferraty, F. and Vieu, P. (2003). Curves discrimination: a nonparametric functional approach. *Computational Statistics & Data Analysis*, 44:161–173.

Simulation results



Probabilities for classification



Estimated coefficient functions with symmetry restrictions