# SQUAREM: An R package for Accelerating Slowly Convergent Fixed-Point Iterations Including the EM and MM algorithms

**Ravi Varadhan**[1*]

1. School of Medicine, Johns Hopkins University
*Contact author: rvaradhan@jhmi.edu

**Keywords:** acceleration, global convergence, polynomial extrapolation, vector extrapolation

Fixed-point iterations are extremely common in applied mathematics. Well known examples include the Jacobi and Gauss-Seidel iterations for solving a linear system of equations, Newton's method for solving a nonlinear system of equations, power method for finding the dominant eigenvector of a matrix, and the expectation maximization (EM) algorithm for finding the maximum likelihood estimate. Fixed-point iterations that are contraction mappings are particularly attractive because of their global convergence property, i.e. they find the fixed-point from any starting point. Many fixed-point iterations are only linearly convergent, and their convergence can be very slow especially when the linear rate constant is close to 1. Over the past several years, we have been working on a class of numerical schemes called squared extrapolation methods (SQUAREM) for accelerating the convergence of smooth, linearly convergent fixed-point iterations (Varadhan 2004; Roland and Varadhan 2005; Roland, Varadhan Frangakis 2007; Varadhan and Roland 2008; and Varadhan 2010 (under preparation)). These numerical acceleration schemes have many attractive properties: (1) they are easy to implement; (2) they are widely applicable to "any" smooth, linearly convergent fixed-point iteration; (3) they utilize minimal memory and computational effort, and therefore, are ideal for high-dimensional problems; and (4) they provide good trade-off between speed of convergence and global convergence.

Here we discuss an R package called SQUAREM that represents the culmination of our research over the past 5 years. This package can accelerate fixed-point iterations under two different situations: (A) when there is an underlying objective function (i.e. a Liapunov function) that is minimized (or maximized) at the fixed-point, or (B) when there is no underlying objective function. Situation A is common in many statistical problems exemplified by the EM and MM (majorize and minimize) algorithms. Situation B is exemplified by fixed-point iterations for solving nonlinear systems and the power method for finding the dominant eigenvector of a matrix. . The package contains 4 main functions which implement different first-order and higher-order SQUAREM schemes for each of the two scenarios. An essential feature of these algorithms is that the user can explicitly choose the degree of trade-off between speed of convergence and global convergence. For example, when reasonable starting values are available or when the fixed-point mapping is globally contractive, the user can opt for more speed by choosing a large value of the non-monotonicity parameter; conversely, the user can opt for more stability at the cost of lesser acceleration by choosing a small value of the non-monotonicity parameter. In our experience, SQUAREM algorithms generally provide good accelerations (at least 2-3 fold) without sacrificing global convergence. In this talk, we will demonstrate the power and utility of SQUAREM on a number of well-known fixed-point iterations.