The Bullwhip Effect under a generalized demand process: an R implementation.

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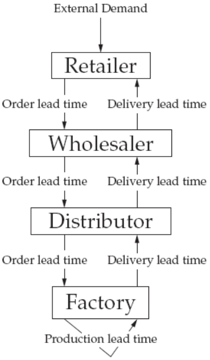
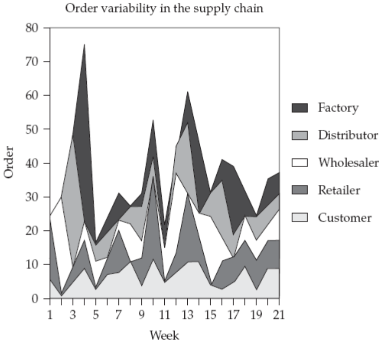
The Bullwhip Effect (BE)

Definition: The BE is the increase of the demand variability as one moves up the supply chain.



The supply chain.



The increase in variability in the supply chain.

Quantifying the BE

A common index used to measure the BE is:

$$M = \frac{Var(q_t)}{Var(d_t)}$$

- $M = 1$, there is no variance amplification.
- $M > 1$, the BE is present.
- $M < 1$, smoothing scenario.

The model

Inventory model

- Two stage supply chain
- Single item with no fixed cost
- OUT replenishment policy
- MMSE as forecast method

Define:

d_t: demand
L: lead time
$y_t = \hat{D}_t^L + z\hat{\sigma}_t^L$
z: $\Phi^{-1}(\alpha)$
$SSLT = z\hat{\sigma}_t^L$

q_t: order quantity
α: the desired SL
$\hat{D}_t^L = \sum_{\tau=1}^{L} \hat{d}_{t+\tau}$
$\hat{\sigma}_t^L = \sqrt{Var(D_t^L - \hat{D}_t^L)}$
$SS = z\sigma_d\sqrt{L}$

$$q_t = y_t - (y_{t-1} - d_t) = (\hat{D}_t^L - \hat{D}_{t-1}^L) + z(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L) + d_t$$

The model

- Demand model, ARMA(p,q)

$$d_t = \mu + \phi_1 d_{t-1} + \cdots + \phi_p d_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q}$$

$$\Phi_p(B) d_t = \mu + \Theta_q(B) \epsilon_t$$

$$\Phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$
$$\Theta_q(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$$

Infinite MA representation of the demand

$$\Phi_p(B)d_t = \mu + \Theta_q(B)\epsilon_t$$

$$d_t = \mu_d + \frac{\Theta_q(B)}{\Phi_p(B)}\epsilon_t = \mu_d + \Psi(B)\epsilon_t$$

where

$\mu_d = \mu/(1 - \phi_1 - \cdots - \phi_p)$ and $\Psi(B) = 1 + \psi_1 B + \psi_2 B^2 + ...$

Recursively calculation

$$\begin{aligned} \psi_j &= \sum_{i=1}^{p} \phi_i \psi_{j-i} \theta_j \\ \psi_0 &= 1,\ \psi_j = 0,\ \text{for } j < 0 \end{aligned}$$

Zhang's (2004) results

The bullwhip effect measure is given by:

$$M = \frac{Var(q_t)}{Var(d_t)} = 1 + \frac{2\sum_{i=0}^{L}\sum_{j=i+1}^{L}\psi_i\psi_j}{\sum_{j=0}^{\infty}\psi_j^2}$$

which implies that there is a bullwhip effect if and only if

$$\sum_{i=0}^{L}\sum_{j=i+1}^{L}\psi_i\psi_j > 0$$

Increasing lead-time exacerbates bullwhip effect if

$$\psi_{L+1}\sum_{j=0}^{L}\psi_j > 0$$

AR(1) case

The AR(1) demand process is described as follow:

$$d_t = \mu + \phi d_{t-1} + \epsilon_t, \ |\phi| < 1$$

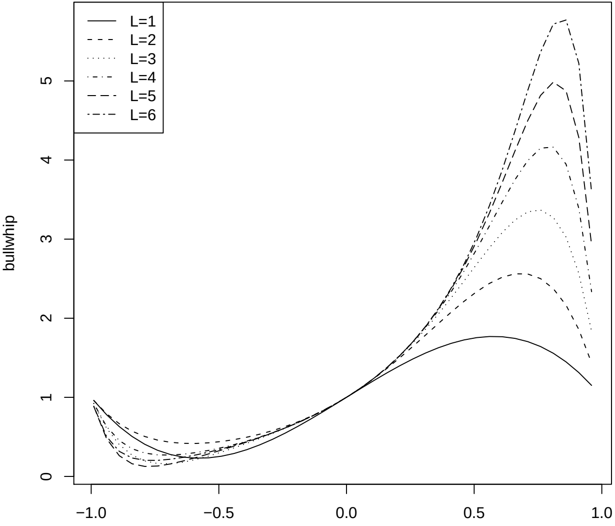
Results:

$$\psi_j = \phi^j, \text{ for } j = 0, 1, 2, ..$$

$$M = 1 + \frac{2\phi(1 - \phi^L)(1 - \phi^{L+1})}{1 - \phi}$$

There is a bullwhip effect if and only if $\phi > 0$.

Figure 1: Relationship between the bullwhip effect and demand autocorrelation



An R implementation: SCperf

Description: Computes the BE and other SC performance variables.

Usage: `SCperf`(ar, ma, *L*, *SL*)

Arguments:

- *ar*: a vector of *AR* parameters,
- *ma*: a vector of *MA* parameters,
- *L*: is the LT plus the review period which is equal to one,
- *SL*: service level, 0.95 by default.

Example:

```
> SCperf(0.95, 0.1, 2, 0.99)
bullwhip   VarD      VarLT    SS        SSLT     z
1.5029     12.3077   5.2025   11.5419   5.3062   2.3264
```

Table1: BE, SS and SSLT generated by SCperf(0.95,0.4,L,0.95)

L	Bullwhip	SS	SSLT
1	1.13711	7.299	1.645
2	1.44321	10.323	4.201
3	1.89270	12.643	7.304
4	2.46294	14.598	10.817
5	3.13393	16.322	14.652
6	3.88802	17.879	18.745
7	4.70970	19.312	23.048
8	5.58531	20.645	27.522
9	6.50289	21.898	32.137
10	7.45199	23.082	36.867

Table 2: SS and SSLT generated by SCperf(0.95,0.4,L,SL)

	L=1		**L=2**		**L=3**	
SL	SS	SSLT	SS	SSLT	SS	SSLT
0.90	5.687	1.282	8.043	3.273	9.850	5.691
0.91	5.950	1.341	8.414	3.424	10.305	5.954
0.92	6.235	1.405	8.818	3.588	10.800	6.239
0.93	6.549	1.476	9.262	3.769	11.343	6.553
0.94	6.899	1.555	9.757	3.971	11.950	6.904
0.95	7.299	1.645	10.323	4.201	12.643	7.304
0.96	7.769	1.751	10.987	4.471	13.456	7.774
0.97	8.346	1.881	11.803	4.803	14.456	8.352
0.98	9.114	2.054	12.889	5.245	15.785	9.120
0.99	10.323	2.326	14.599	5.941	17.881	10.330

Conclusions

- `SCperf` overcomes the difficulty of calculate the BE thanks to the help of `ARMAtoMA` function.
- The use of `SCperf` makes possible to get accurate estimations of the BE and other SC performance variables.
- For certain types of demand processes the use of MMSE leads to significant reduction in the safety stock level.
- `SCperf` leads to a simple but powerful tool which can be helpful for the study of SCM research problems.
- `SCperf` might be used to complement other managerial support decision tools.

References:

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- Zhang, X., 2004a. *The impact of forecasting methods on the bullwhip effect.*International Journal of Production Economics Vol 88 No 1, 15-27.
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