Monte Carlo Simulation for Pricing European and American Basket option

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Accurate and simple pricing of basket options of European and American style can be a daunting task. Several approaches have been proposed in the literature to price path-dependent derivatives of European or American kind. They can be categorized as follows:

1) analytical approximations;
2) tree based methods;
3) Monte Carlo simulations;

Examples of the analytical approximations are provided in Milevsky and Posner 1998 [3] and [4] who compare the relative accuracy of the lognormal or the inverse gamma distribution for approximating the sum of lognormal distributions. Tree based methods were originally proposed by Cox et al 1979 [2] and adopted in Wan 2002 [5]. Monte Carlo methods were first proposed by Boyle 1977 [1] as an alternative to the closed form solution of a partial differential equation or the use of tree based methods. Monte Carlo methods can be fruitfully used to price derivatives lacking an analytical closed-form. In the more general setup, assuming a deterministic risk-free interest rate, the value of an option of European kind is given by the following expectation

$$P_t = e^{\int_t^T (-r(T-\tau)d\tau)} E_Q[g(T, S)] \tag{1}$$

In the paper we focus on the problem of pricing options on the following portfolio composed by n correlated stocks:

$$V_t = \Sigma_{i=1}^{n} a_i \cdot S_i \tag{2}$$

The pool of stocks follow a system of geometric brownian motion (GBM):

$$\begin{aligned} \frac{dS_1}{S_1} &= rdt + \sigma_1 dw_1 \\ \frac{dS_2}{S_2} &= rdt + \sigma_2 dw_2 \\ \frac{dS_n}{S_n} &= rdt + \sigma_n dw_n \end{aligned} \tag{3}$$

where r is the continuosly compounding risk-less interest rate, σ_i and dw_i are respectively the return volatility and the standard brownian motion driving the asset S_i. The different brownian motions are generally correlated with a given correlation matrix $\Omega = E[dw' \cdot dw]$. The Cholesky decomposition of the correlation matrix is adopted to build a recursive system for the equations of each stock composing the portfolio. Once we have computed M sample paths for our stocks we can obtain M sample path for the value of the whole portfolio. At this point we are able to evaluate the payoff function $g^s(T, V)$ for each replication s . Finally the option price will be approximated by the following mean and standard deviation:

$$\begin{aligned} \hat{C} &= \frac{1}{M} \Sigma_{s=1}^{M} e^{(-r(T-t))} g^s(T, V) \\ \sigma_{\hat{C}} &= \sqrt{\frac{1}{M} \Sigma_{s=1}^{M} (C_s - \hat{C})^2} \end{aligned} \tag{4}$$

This article shows the numerical features of a suite of R functions aimed at pricing European and American style basket options. The performances of these R functions are compared with a simulation engine equipped with an automatic FORTRAN translator for the equations describing the portfolio composition. The R functions are tested against three different sizes correlated multiasset options. Two different variance reduction techniques are also explored. In the models we have examined, the comparison among the different methods has shown a definite superiority of the control variates techniques in reducing the estimating variance. Code vectorization is going to be carried out for performance improvements

References

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