Portfolio Optimization with R/Rmetrics

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Portfolio Optimization Problem

… return, risk, performance ratio

For a given set of financial assets let us find the composition

1) which minimizes the risk for a given return (reward),

2) which maximizes the return for a given risk,

3) which optimizes a reward/risk performance ratio,

4) which finds the global minimum risk,

subject to certain **constraints and preferences**.

How to quantify Risk ?

Stone 1973

$$R_S[Y_0, k, A](f) = \left(\int_{-\infty}^{A} |y - Y_0|^k f(y)\, dy \right)^{1/k}$$

$$R_{SD}(f) = R_S[\mu_y, 2, \infty](f) \qquad R_{SSD}(f) = R_S[\mu_y, 2, 0](f)$$

$$R_{SVM}(f) = R_S[\mu_y, 2, 0]^2(f) \qquad R_{\alpha-t}(f) = R_S[t, \alpha, t]^{\alpha}(f)$$

y are the financial returns,
f ( ) their multivariate distribution
A, Y_0, and k parameters

Includes:
Mean - Covariance Risk
Mean - CVaR Measure:
k = 1, A = VaR, Y_0 = 0

Pederson and Satchell 1998

$$R[A,\ b,\ \alpha,\ \theta,\ W(\cdot)] = \left[\int_{-\infty}^{A} |y - b|^{\alpha} W[F(y)] f(y)\, dy \right]^{\theta}$$

for some bounded function W ( )

(*BP*1) (*Nonnegativity*): $R[\tilde{y}] \geq 0$.
(*BP*2) (*Homogeneity*): $R[\lambda\tilde{y}] = |\lambda|\, R[\tilde{y}]$ *for* $\lambda \geq 0$.
(*BP*3) (*Subadditivity*): $R[\tilde{y}_1 + \tilde{y}_2] \leq R[\tilde{y}_1] + R[\tilde{y}_2]$.
(*BP*4) (*Shift-invariance*): $R[\tilde{y} + \lambda] \leq R[\tilde{y}]$ *for all* λ.

Artzner, Delbaen, Eber, Heath 1999

(ADEH 3) (translation invariance) R(X + c) = R(X) - c for all c

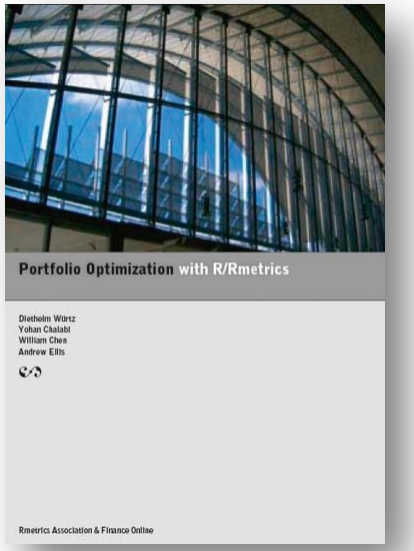
(ADEH 4) (monotonicity) X ≤ Y ⇒ R(Y) ≤ R(X).

… this makes a coherent risk measure

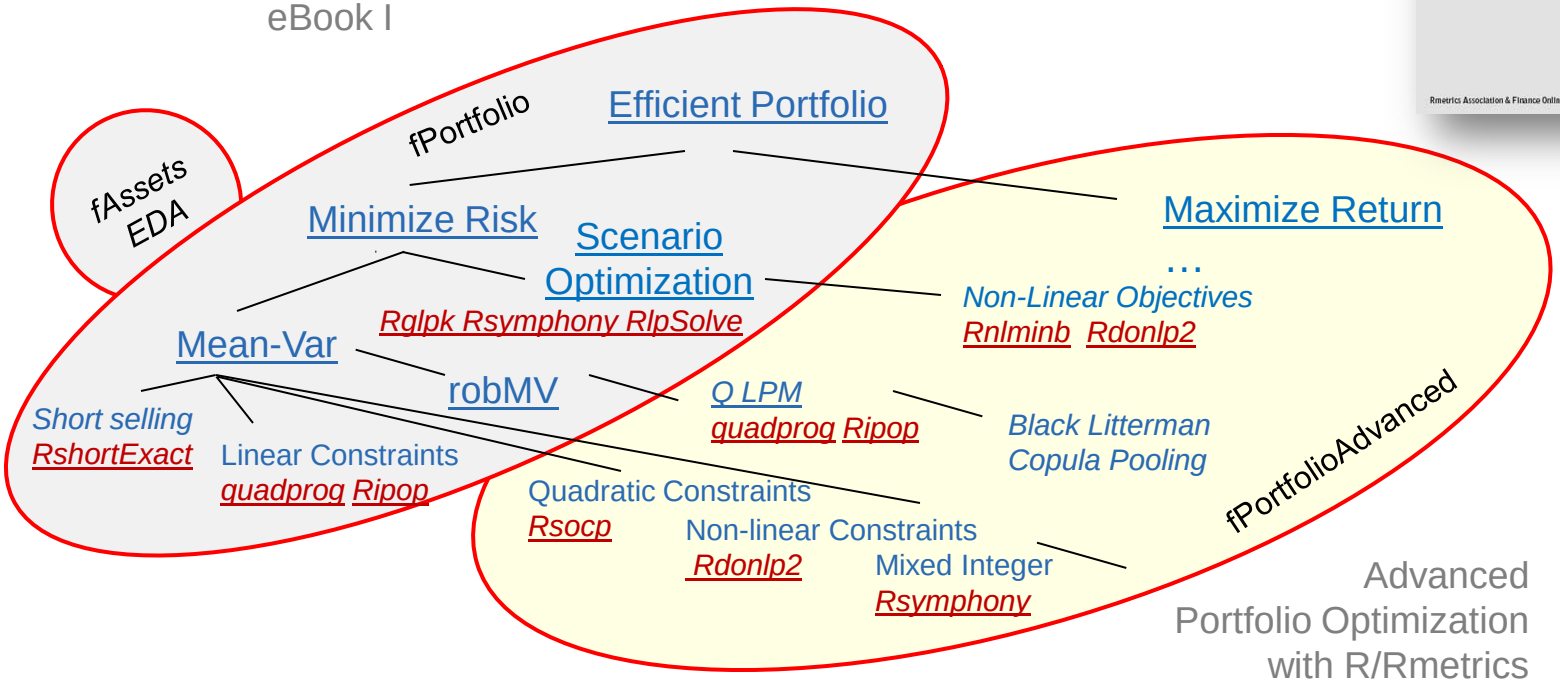
fPortfolio Models

fPortfolio Zoo:
Rmetrics Software

Topics
Managing Data Sets of Assets
Exploratory Data Analysis of Assets
Portfolio Framework
Mean-Variance Portfolios
Mean-CVaR Portfolios
Portfolio Backtesting



Portfolio Optimization
with R/Rmetrics
eBook I



Advanced
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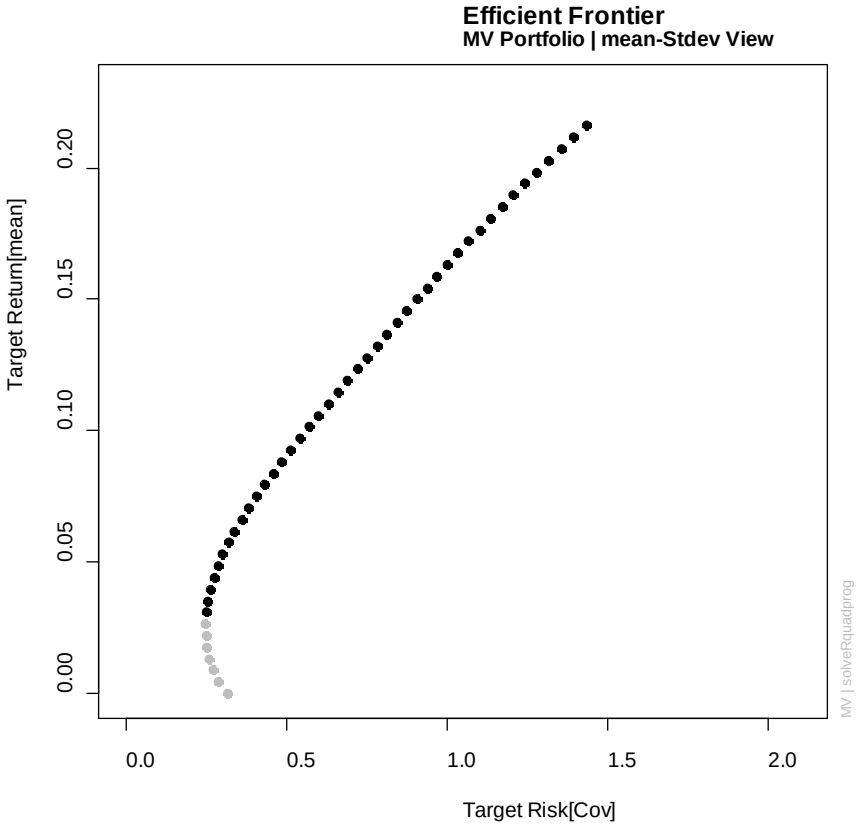
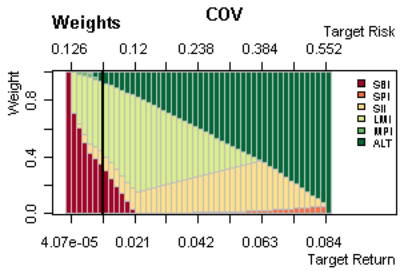
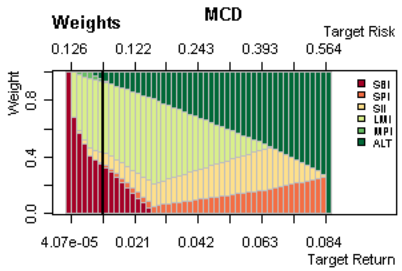
Plotting Frontiers and Weights ...

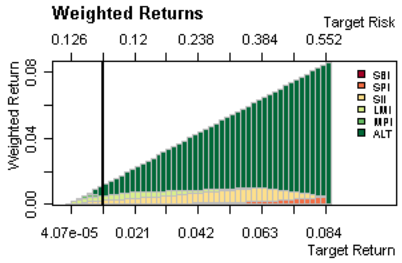


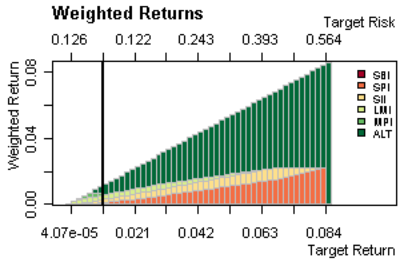
Reducing Estimation Errors ...

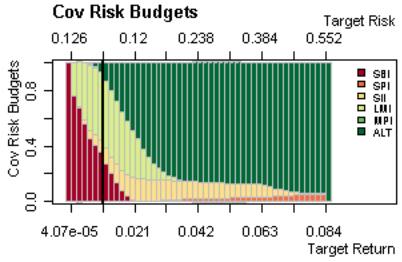
```
Functions:
covEstimator
kendallEstimator
spearmanEstimator
mcdEstimator
mveEstimator
covMcdEstimator
covOGKEstimator
shrinkEstimator
baggedEstimator
```

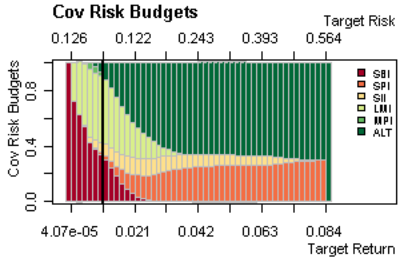


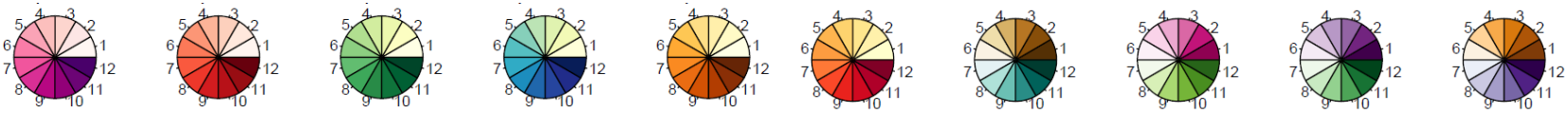






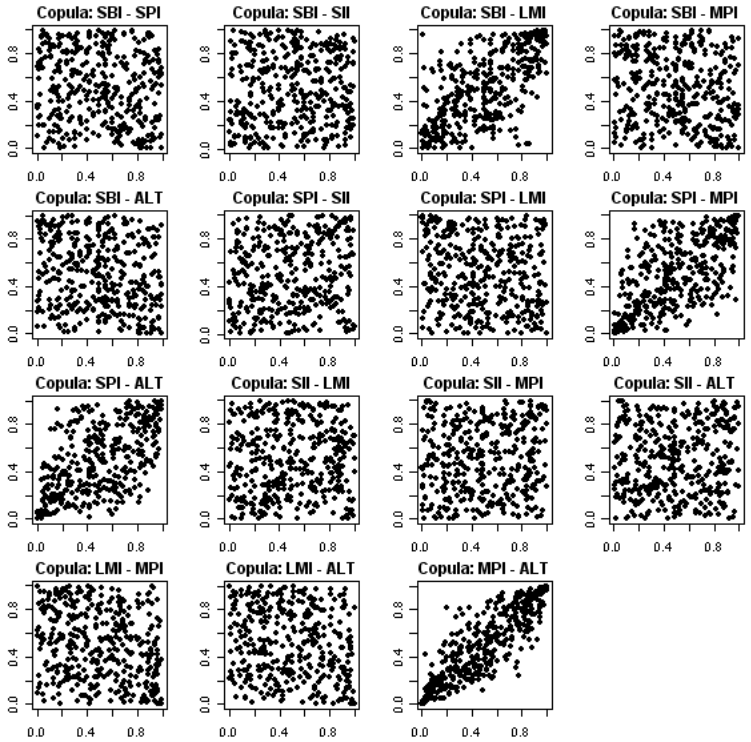






Copulae Tail Risks ...

Copulae Lower Tail Risk Dependence Budgets



SBI CH Bonds
SPI CH Stocks
SII CH Immo
LMI World Bonds
MPI World Stocks
ALT World AltInvest

Tail Dependence:
Lower

$$\lambda_{lower} = \lim_{u \to 0} \left[\Pr \left(Y \le F_Y^{-1}(u) \mid X \le F_X^{-1}(u) \right) \right] = \lim_{u \to 0} \left[\frac{C(u,u)}{u} \right]$$

$$\begin{aligned} \min \quad & w^\top \hat{\Sigma} \, w \\ s.\ t. \quad & \\ & w^\top \hat{\mu} = \bar{r} \\ & w^\top 1 = 1 \\ \mathcal{L}_i^{lower} \le & \frac{w_i}{\lambda} \frac{d\lambda}{dw_i} \le \mathcal{L}_i^{upper} \\ & \dots \end{aligned}$$

... Quadratic Constraints use Rsocp
(not yet fully implemented)

GCC Country Rotation