Financial econometrics based on stochastic differential equations and the sde package

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Keywords: financial econometrics, quasi-likelihood analysis, stochastic differential equations, simulation, mathematical finance

In this talk we will introduce the package `sde` which contains generic functions for simulation and inference on stochastic differential equations. In particular, stochastic differential equations corresponding to diffusion processes driven by the Wiener process are considered.

Most of the theoretical results in modern finance rely on the assumption that the underlying dynamics of asset prices, currencies exchange rates, interest rates, etc are continuous time stochastic processes driven by stochastic differential equations. Continuous time models are also at the basis of option pricing and option pricing often requires Monte Carlo methods. In turn, the Monte Carlo method requires a preliminary good model to simulate whose parameters has to be estimated from the data. On the other side, most applications in financial econometrics make use of pure time series modeling because many statistical procedures are already available in many statistical packages.

The discrepancy between theoretical and applied mathematical finance is motivated by the fact that while the model is continuous, the observations always come in discrete time. Inference for continuous time data from stochastic differential equation dates back to Jacod and Shiryayev (1987) and today is considered as a solved problem. On the contrary, the likelihood function for discretized stochastic differential equations is available only for a very limited class of models and exact likelihood inference is usually not possible. Also, discretization of the estimators obtained from continuous time analysis is always biased and not useful in practice.

Recently, many authors have considered ways to establish approximate and/or quasi-likelihood inference for stochastic differential equations (for a review see Iacus, 2008). The `sde` package implements those methods in the hope to fill the gap between theoretical results and applied financial econometrics. In particular, the package allow to build several kinds of likelihood functions to be used in a standard `R` context via the `mle` function.

The `sde` package also implements model selection procedures based on AIC statistics for stochastic differential equations, identification of structural changes in the volatility component of the model and hypotheses testing along with other estimation procedures like estimating functions, the method of the moments, etc. Some tools for nonparametric statistics are also available.

Due to the fact that simulation is part of modern financial analysis, the `sde` package includes the function `sde.sim` which implements several simulation schemes for one dimensional stochastic differential equations, including those presented in the fundamental reference of Kloden and Planten (1999), e.g. Euler's and both Milstein's schemes, as well as several new simulation methods appeared in the last ten years, e.g. Ozaki and Shoji-Ozaki local linearization methods, Berkos et al. Exact Sampling, and Kloden-Platen-Soeresen method.

References

Iacus. S.M. (2008). Simulation and Inference for Stochastic Differential Equations with R Example, Springer, New York.

Jacod, J., Shiryayev, A.N. (1987) Limit Theorems for Stochastic Processes, Springer-Verlag, New York.

Kloden, P., Platen, E. (1999). Numerical Solution of Stochastic Differential Equations, Applied Mathematics, **23**, Third corrected printing, Springer, New York.