Automated Modeling and Forecasting Vector Autoregressive Processes

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Agenda

1. Vector Autoregressive Processes

2. VAR Modeling with AuFVAR
   Data Initial Analysis
   Model Settings Selection
   Structural Breaks
   Estimation and Forecasting
   Residual Analysis

3. Empirical Example: Advertisement Spendings

VAR Model: Theoretical Basics

- **VAR(p) model for k-variate process Y_t:**
  $$Y_t = \nu + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t.$$
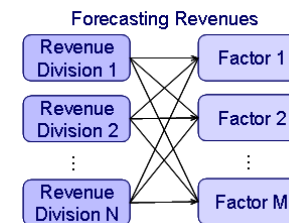
- **Standard Modeling Steps:**
  1. Identification of relevant variables and data initial analysis
  2. Lag order selection
  3. Parameter estimation for selected model
  4. Forecasting
  5. Residual analysis
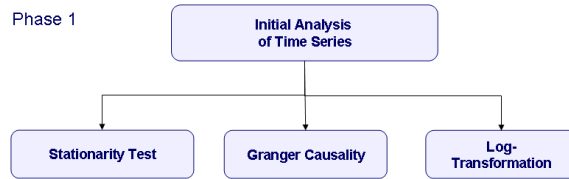

Special Characteristics of AuFVAR

1. Incorporation of time trend and season as exogenous variables:
   $$Y_t = \nu + \underbrace{n_{1t}\nu_1 + \ldots + n_{1(s-1)}\nu_{s-1}}_{\text{Season Dummies}} + \underbrace{\gamma t_t}_{\text{Time Trend}} + A(L)Y_t + \varepsilon_t.$$

2. Definition of a VAR model with n time series in each model

3. Automated model structure selection (settings und lag order) by means of the information criteria and backtesting

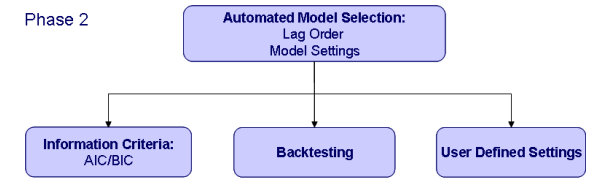4. Repeated forecasts for different VAR models with similar time series structure

Forecasting Revenues

Revenue Division 1 → Factor 1
Revenue Division 2 → Factor 2
⋮
Revenue Division N → Factor M

Data Initial Analysis in AuFVAR

Phase 1

```
Initial Analysis
of Time Series
    │
    ├────────────┬──────────────┐
    ▼            ▼              ▼
Stationarity    Granger      Log-
Test            Causality    Transformation
```

1. Stationarity Test: ADF Test
   - Function: *adf.test* (modified)
2. Granger Causality Test
   - Function: *grangertest* (standard)
3. Log Level Transformation
   - Natural Logarithm of Time Series

Model Selection

Phase 2

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Automated Model Selection:
Lag Order
Model Settings
    │
    ├────────────┬──────────────┐
    ▼            ▼              ▼
Information     Backtesting   User Defined Settings
Criteria:
AIC/BIC
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Definition the different VAR models (with/without trend/season, lag order variation) and selection the model with the least criterion value:
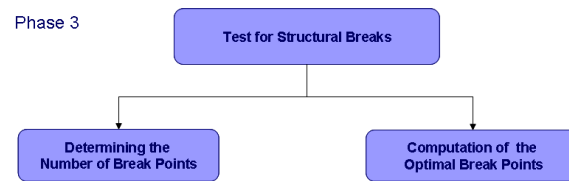
1. Information Criteria, e.g.
$$AIC(p) = \ln \det(\widetilde{\Sigma}_\varepsilon(p)) + \frac{2}{T} pK^2,$$
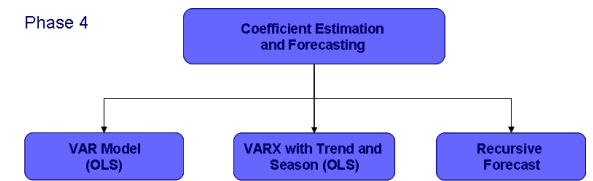2. Backtesting (out-of-sample):
$$RMSE = \sqrt{\frac{1}{h} \sum_{i=1}^{h} (y_{T+h} - y_t(h))^2}.$$

Test for Structural Breaks

Phase 3

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Test for Structural Breaks
    │
    ├────────────────────┐
    ▼                    ▼
Determining the      Computation of the
Number of Break      Optimal Break Points
Points
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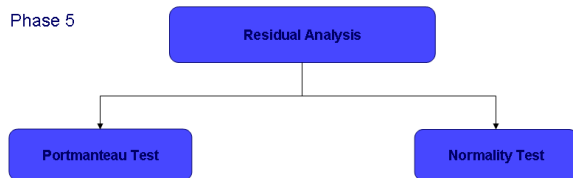
- Testing for the structural breaks:
  Function *breakpoints*, library *strucchange*
  1. Determining the number of breakpoints
  2. Computation of the optimal breakpoints
  3. Adjustment of the model definition

Estimation and Forecasting

Phase 4

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Coefficient Estimation
and Forecasting
    │
    ├────────────┬──────────────┐
    ▼            ▼              ▼
VAR Model     VARX with Trend  Recursive
(OLS)         and Season (OLS) Forecast
```

1. Estimation of the appropriate model after testing for structural breaks
   - Function: *estVARXls* for the models with intercept and lagged variables (standard)
   - Function: *estVARXlsM* for the models with additional time trend and/or season (modified)
2. Forecasting the time series, computation of confidence bounds and corresponding plots

Residual Analysis

Phase 5

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Residual Analysis
   │
   ├──────────────────┐
   ▼                  ▼
Portmanteau Test   Normality Test
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1. Portmanteau Test for Autocorrelation:

   - The standard test statistic, if time series length $T \geq 100$
   - The modified test statistic otherwise

2. Test for Nonnormality based on Skewness and Kurtosis (Lütkepohl, 1993)

Example: Time Series and Forecasting

Data

- Variables to be forecasted:
  - Common Advertisement Earnings (with/without media)
- Further possible variables for the VAR model:
  1. ZEW Index (Centre for European Economic Research)
  2. Incoming Orders (Germany)
  3. CDAX (Composite DAX)

Common Advertisement Earnings
Forecast for July 2005 until December 2006