Testing Volatility Interactions in a Constant Conditional Correlation GARCH Model

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Introduction and Motivation

The Constant Conditional Correlation (CCC) GARCH model

- ✔ One of the most popular MGARCH models
- ✔
  - ✘
  - ✘
- ✔
  - ✘
  - ✘

Introduction and Motivation

* Intro & Motivation
* Modifications
* Bivariate Models
* LM Test
* Two Constraints
* Constraint for $\mathbf{h}_t$
* Empirical Example
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Two Directions for Modification

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1. **Changing Conditional Correlations**
   - ✔ Dynamic Conditional Correlation
   - ✔ Smooth Transition Conditional Correlation
2. **Interactions in Volatilities Among Assets**
   - ✔ Extended CCC

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We take the 2nd direction.

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The Bivariate $(1, 1)$ Models

The CCC-GARCH Model

The Extended CCC-GARCH Model

The Bivariate $(1,1)$ Models

The CCC-GARCH Model

$$\mathbf{h}_t = \begin{bmatrix} h_{1,t} \\ h_{2,t} \end{bmatrix} = \mathbf{a}_0 + \mathbf{A}_1 \boldsymbol{\varepsilon}_{t-1}^{(2)} + \mathbf{B}_1 \mathbf{h}_{t-1}$$
$$= \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix}$$

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The Lagrange Multiplier Test

✔ A Test for Volatility Interaction

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  - ✘ H_0 is Often Rejected in Real Data
  - ✘ Need to Estimate ECCC-GARCH Model

Two Types of Constraints

There are two types of constraints for:

- ✔
- ✔

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- ✔ Positive definiteness of covariance matrix:

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$$\lambda(\mathbf{A}_1 + \mathbf{B}_1) < 1$$

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a trivial sufficient condition (elementwise):

$$\mathbf{a}_0 > \mathbf{0}, \quad \mathbf{A}_1 \geq \mathbf{0} \quad \text{and} \quad \mathbf{B}_1 \geq \mathbf{0}$$

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The second condition can be relaxed!

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Constraints for Positive $\mathbf{h}_t$

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The ECCC-GARCH$(1,1)$ Model in the ARCH(∞) form

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$$\begin{aligned}\mathbf{h}_t &= \mathbf{a}_0 + \mathbf{A}_1\boldsymbol{\varepsilon}_{t-1}^{(2)} + \mathbf{B}_1\mathbf{h}_{t-1} \\ &= (\mathbf{I}_N - \mathbf{B}_1)^{-1}\mathbf{a}_0 + \sum_{j=1}^{\infty}\mathbf{B}_1^{k-1}\mathbf{A}_1\boldsymbol{\varepsilon}_{t-j}^{(2)}\end{aligned}$$

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Three constraints for positive $\mathbf{h}_t$: (elementwise)

(1) $(\mathbf{I}_N - \mathbf{B}_1)^{-1}\mathbf{a}_0 > \mathbf{0}$
(2) $\mathbf{A}_1 \geq \mathbf{0}$
(3) $\mathbf{B}_1^{k-1}\mathbf{A}_1 \geq \mathbf{0}$ for all $k \in \mathbb{N}$.

Due to (3), able to have negative coefficients in $\mathbf{B}_1$.
But seems difficult to code it!

Empirical Example

Partial Results of Bivariate GARCH Models

Model	Stock	$\mathbf{B}_1$		−LogLik	LM
CCC	NEC	0.910		**23833.11**	**82.97**
		(0.009)			[0.00]
	Toshiba		0.898		
			(0.011)		

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ECCC w. C	NEC	0.888	**1E-8**	**23799.10**	
		(0.011)	**(9E-6)**		
	Toshiba	**7E-8**	0.869		
		(2E-5)	(0.013)		

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ECCC w.o. C	NEC	0.897	**-0.033**	**23789.37**	
		(0.017)	**(0.010)**		
	Toshiba	**-0.031**	0.900		
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The estimated $\mathbf{B}_1$ does not satisfy $\mathbf{B}_1^{k-1}\mathbf{A}_1 \geq \mathbf{0}$ for all $k \in \mathbb{N}$.

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  - ✘ ending up with negative off-diagonal elements

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