KernGPLM – A Package for Kernel-Based Fitting of Generalized Partial Linear and Additive Models

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Aim of this Talk

analysis of highdimensional data by semiparametric (generalized) regression models

- compare different approaches to additive models (AM) and generalized additive models (GAM)
- include categorical variables $\Longrightarrow$ *partial linear* terms (combination of AM/PLM and GAM/GPLM)
- provide software $\Rightarrow$ R package **KernGPLM**
- focus on kernel-based techniques for **high-dimensional data**

Financial application: Credit Rating

- new interest in this field because of **Basel II**:
  capital requirements of a bank are adapted to the individual credit portfolio
- key problems: determine **rating score** and subsequently **default probabilities (PDs)** as a function of some explanatory variables

$\rightarrow$ classical **logit/probit-type models** to estimate linear predictors (scores) and probabilities (PDs)

Two objectives:

- study **single factors**
- find the **best model**

Binary choice model

$\rightarrow$ credit rating: estimate scores + PDs

$$P(Y=1|\boldsymbol{X}) = E(Y|\boldsymbol{X}) = G(\boldsymbol{\beta}^\top \boldsymbol{X})$$

$\rightarrow$ parametric binary choice models

logit	$P(Y=1\|\boldsymbol{X}) = F(\boldsymbol{X}^\top\boldsymbol{\beta})$	$F(\bullet) = \frac{1}{1+e^{-\bullet}}$
probit	$P(Y=1\|\boldsymbol{X}) = \Phi(\boldsymbol{X}^\top\boldsymbol{\beta})$	$\Phi(\bullet)$ standard normal cdf

Generalized linear model (GLM)

$$E(Y|\boldsymbol{X}) = G\left(\boldsymbol{X}^\top\boldsymbol{\beta}\right)$$

Data Example: Credit Data

References: Fahrmeir/Hamerle (1984); Fahrmeir & Tutz (1995)

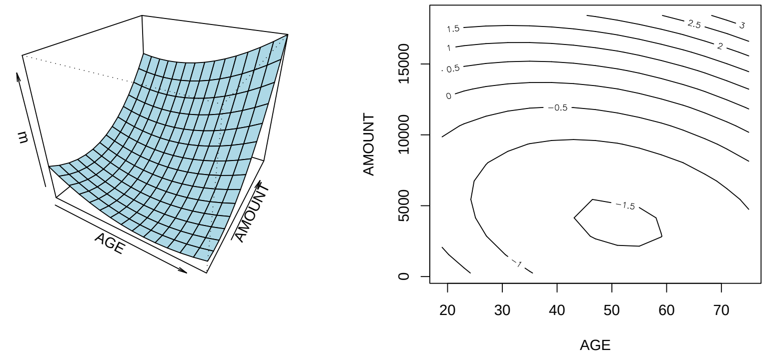
- default indicator: $Y \in \{0,1\}$, where $1 =$ default
- explanatory variables:
  personal characteristics, credit history, credit characteristics
- sample size: 1000 (stratified sample with 300 defaults)

Estimated (Logit) Scores

$$\begin{aligned}\text{Score} \quad = \quad & 1.334 - 0.763^{***}\cdot \text{previous} - 0.310\cdot \text{employed} + 0.566^{**}\cdot (\text{d9-12}) \\ & +0.898^{**}\cdot (\text{d12-18}) + 0.981^{***}\cdot (\text{d18-24}) + 1.550^{***}\cdot (\text{d>24}) \\ & -0.984^{***}\cdot \text{savings} - 0.363^{**}\cdot \text{purpose} + 0.660^{***}\cdot \text{house} \\ & -0.000251^{**}\cdot \text{amount} - 0.0942^{**}\cdot \text{age} + 0.0000000173^{**}\cdot \text{amount}^2 \\ & +0.000833^{*}\cdot \text{age}^2 + 0.00000236\cdot (\text{amount} \cdot \text{age})\end{aligned}$$

*, **, *** denote significant coefficients at the 10%, 5%, 1% level, respectively

Data Example: Logit (with interaction)



credit default on AGE and AMOUNT using quadratic and interaction terms, left: surface and right: contours of the fitted score function

Semiparametric Models

- local regression

$$E(Y|\boldsymbol{T}) = G\{m(\boldsymbol{T})\}, \quad m \text{ nonparametric}$$

- generalized partial linear model (GPLM)

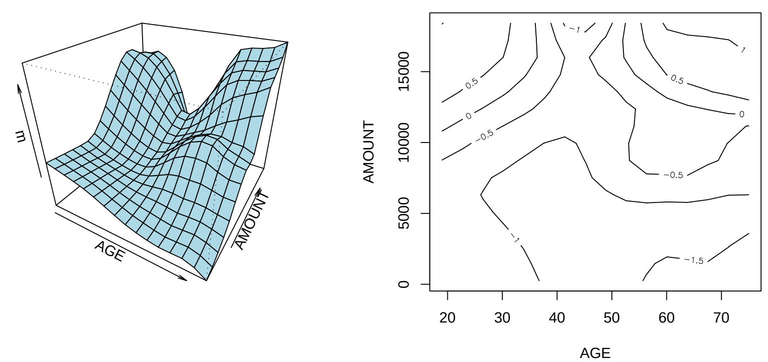
$$E(Y|\boldsymbol{X},\boldsymbol{T}) = G\left\{\boldsymbol{X}^\top\boldsymbol{\beta} + m(\boldsymbol{T})\right\} \quad m \text{ nonparametric}$$

- generalized additive partial linear model (semiparametric GAM)

$$E(Y|\boldsymbol{X},\boldsymbol{T}) = G\left\{\beta_0 + \boldsymbol{X}^\top\boldsymbol{\beta} + \sum_{j=1}^{p} m_j(T_j)\right\} \quad m_j \text{ nonparametric}$$

Some references:
Loader (1999), Hastie and Tibshirani (1990), Härdle et al. (2004), Green and Silverman (1994)

Data Example: GPLM



credit default on AGE and AMOUNT using a nonparametric function, left: surface and right: contours of the fitted score function on AGE and AMOUNT

Estimation Approaches for GPLM/GAM

- GPLM:
  - ⋆ generalization of Speckman's estimator (type of profile likelihood)
  - ⋆ backfitting for two additive components and local scoring

References:
(PLM) Speckman (1988), Robinson (1988); (PLM/splines) Schimek (2000), Eubank et al. (1998), Schimek (2002); (GPLM) Severini and Staniswalis (1994), Müller (2001)

- semiparametric GAM:
  - ⋆ [modified | smooth] backfitting and local scoring
  - ⋆ marginal [internalized] integration

References:
(marginal integraton) Tjøstheim and Auestad (1994), Chen et al. (1996), Hengartner et al. (1999), Hengartner and Sperlich (2005);
(backfitting) Buja et al. (1989), Mammen et al. (1999), Nielsen and Sperlich (2005)

Estimation of the GPLM: generalized Speckman estimator

- partial linear model (identity G)

$$E(Y|\boldsymbol{X},\boldsymbol{T}) = \boldsymbol{X}^T\boldsymbol{\beta} + m(\boldsymbol{T})$$

$$\Longrightarrow \quad \begin{aligned} \boldsymbol{m}^{new} &= \mathbf{S}(\boldsymbol{Y} - \mathcal{X}\boldsymbol{\beta}) \\ \boldsymbol{\beta}^{new} &= (\widetilde{\mathcal{X}}^T\widetilde{\mathcal{X}})^{-1}\widetilde{\mathcal{X}}^T\widetilde{\boldsymbol{Y}} \end{aligned}$$

- generalized partial linear model

$$E(Y|\boldsymbol{X},\boldsymbol{T}) = G\{\boldsymbol{X}^T\boldsymbol{\beta} + m(\boldsymbol{T})\}$$

$\Longrightarrow$ above for adjusted dependent variable

$$\boldsymbol{Z} = \mathcal{X}\boldsymbol{\beta} + \boldsymbol{m} - \mathcal{W}^{-1}\boldsymbol{v},$$

$\boldsymbol{v} = (\ell_i')$, $\mathcal{W} = \mathrm{diag}(\ell_i'')$

References: Severini and Staniswalis (1994)

Comparison of Algorithms

	parametric step	nonparametric step	est. matrix
Speckman	$\boldsymbol{\beta}^{new} = (\widetilde{\mathcal{X}}^T\mathcal{W}\widetilde{\mathcal{X}})^{-1}\widetilde{\mathcal{X}}^T\mathcal{W}\widetilde{\boldsymbol{Z}}$	$\boldsymbol{m}^{new} = \mathbf{S}(\boldsymbol{Z} - \mathcal{X}\boldsymbol{\beta})$	$\boldsymbol{\eta} = \mathcal{R}^S\boldsymbol{Z}$
Backfitting	$\boldsymbol{\beta}^{new} = (\mathcal{X}^T\mathcal{W}\widetilde{\mathcal{X}})^{-1}\mathcal{X}^T\mathcal{W}\widetilde{\boldsymbol{Z}}$	$\boldsymbol{m}^{new} = \mathbf{S}(\boldsymbol{Z} - \mathcal{X}\boldsymbol{\beta})$	$\boldsymbol{\eta} = \mathcal{R}^B\boldsymbol{Z}$
Profile	$\boldsymbol{\beta}^{new} = (\mathcal{X}^T\mathcal{W}\widetilde{\mathcal{X}})^{-1}\mathcal{X}^T\mathcal{W}\widetilde{\boldsymbol{Z}}$	$\boldsymbol{m}^{new} = \ldots$	$\boldsymbol{\eta} = \mathcal{R}^P\boldsymbol{Z}$

Speckman/Backfitting:
$\widetilde{\mathcal{X}} = (\mathbf{I} - \mathbf{S})\mathcal{X}$, $\widetilde{\boldsymbol{Z}} = (\mathbf{I} - \mathbf{S})\boldsymbol{Z}$, $\mathbf{S}$ weighted smoother matrix

Profile Likelihood:
$\widetilde{\mathcal{X}} = (\mathbf{I} - \mathbf{S}^P)\mathcal{X}$, $\widetilde{\boldsymbol{Z}} = (\mathbf{I} - \mathbf{S}^P)\boldsymbol{Z}$, $\mathbf{S}^P$ weighted (different) smoother matrix

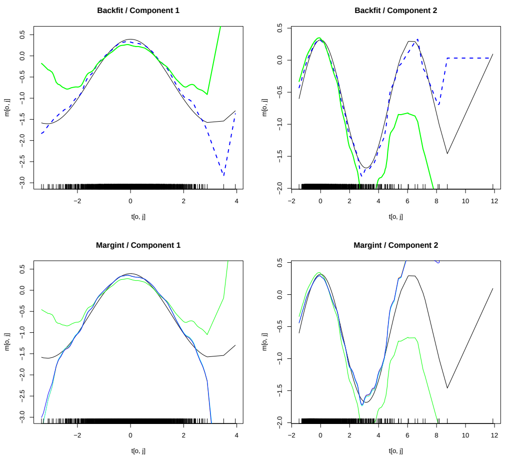
References: Severini and Staniswalis (1994), Müller (2001)

Estimation of the GAM

$$E(Y|\boldsymbol{X},\boldsymbol{T}) = G\left\{\beta_0 + \boldsymbol{X}^\top\boldsymbol{\beta} + \sum_{j=1}^{p} m_j(T_j)\right\} \qquad m_j \text{ nonparametric}$$

- **classical backfitting:** fit single components by regression on the residuals w.r.t. the other components
- **modified backfitting:** first project on the linear space spanned by all regressors and then nonparametrically fit the partial residuals
- **marginal (internalized) integration:** estimate the marginal effect by integrating a full dimensional nonparametric regression estimate
  $\Longrightarrow$ original proposal is computationally intractable: $O(n^3)$
  $\Longrightarrow$ choice of nonparametric estimate is essential: **marginal internalized integration**

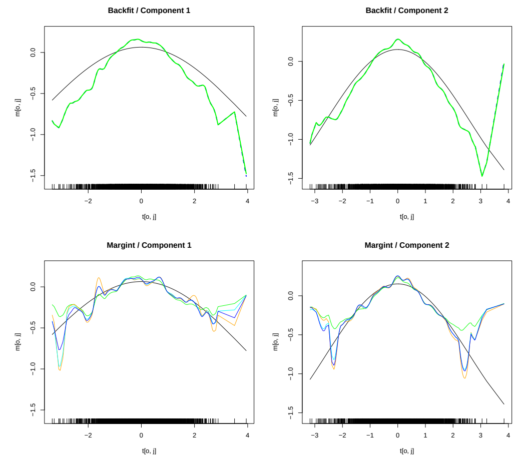
Simulation Example: True Additive Function



- B – classical
- B – modified
- M – classical
- M – pdf estimate 1
- M – pdf estimate 2
- M – normal pdfs

Marginal integration – as initialization for backfitting

Simulation Example: True Non-Additive Function



- B – classical
- B – modified
- M – classical
- M – pdf estimate 1
- M – pdf estimate 2
- M – normal pdfs

Marginal integration – estimate of marginal effects

Comparison of Algorithms

- consistency of marginal integration:
  - ⇒ if underlying function is truly additive, backfitting outperforms marginal integration
  - ⇒ consider marginal integration to initialize backfitting (replacing the usual zero-functions)
- comparison of backfitting and marginal integration:
  - ⇒ marginal integration indeed estimates marginal effects, but large number of observations is needed
  - ⇒ estimation method of the instruments is essential, dimension reduction techniques are required

Summary

- GPLM and semiparametric GAM are natural extensions of the GLM
- *large amount of data* is needed for estimating marginal effects

⇒ R package **KernGPLM** with routines for
  - ⋆ (kernel based) generalized partial linear and additive models
  - ⋆ additive components by [modified] backfitting + local scoring
  - ⋆ additive components by marginal [internalized] integration

- possible extensions:
  - ⋆ smooth backfitting
  - ⋆ externalized marginal integration

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