Robustness Assessment for Composite Indicators

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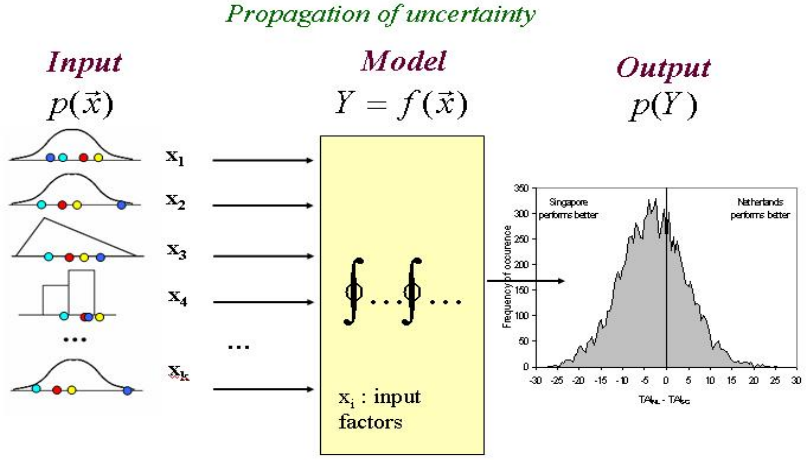
Introduction

- Objectives of Sensitivity Analysis (examples):
  - Help identify key sources of variability (to assist policy making, risk management strategy)
  - Help identify key sources of uncertainty (to prioritize additional data collection to reduce uncertainty)
  - Variance of an output
  - What causes worst/best outcomes
  - What are critical control points, critical limits
- Local vs. Global Sensitivity Analysis
- Model Dependent vs. Model Independent Sensitivity Analysis
- Applicability of methods often depends upon characteristics of a model (e.g., nonlinear, thresholds, categorical inputs, etc.)

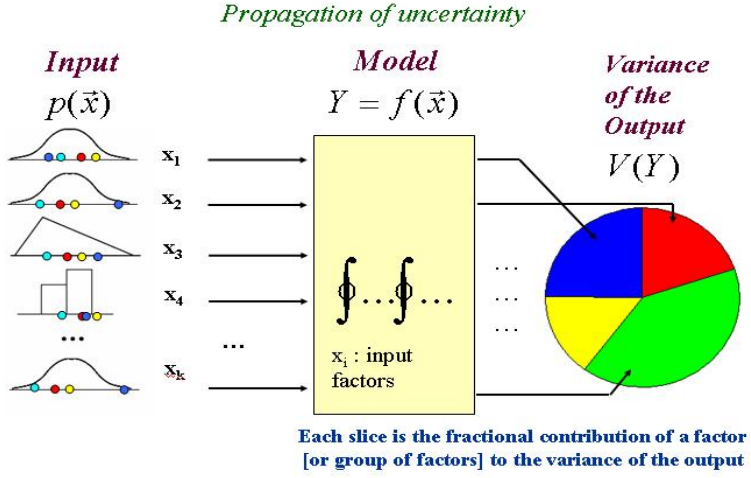
Moving from *Uncertainty Analysis*



Uncertainty Analysis UA (Janssen, RIVM, The Netherlands):

The study of the uncertain aspects of a model and of their influence on the (uncertainty of the) model output

to *Sensitivity Analysis*



Sensitivity Analysis SA (Saltelli, EU JRC, Ispra):

The study of how the uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input

Ideal SA Method

- **Cope with scale and shape of the input factors:** Range of the factor variation and shape / parameters of the pdf.
- **Include multi-dimensional averaging:** Global versus local methods
- **Model independent (model free):** Cope with non-linear / non-additive, non-monotonic models
- **Grouping of factors:** Treat grouped factors as if they were single factors Cost efficient Pay attention to computational costs C

SA types

- Local or global
- Qualitative or quantitative

Sobol' Sensitivity Measures

First-order Sensitivity Measure (S_i)

Measures the fractional contribution of x_i to the variance of $f(\mathbf{x})$ without accounting for interactions of x_i with the other factors.

$$S_i \equiv \frac{V_{x_i}\left(E_{\mathbf{x}_{-i}}(Y|x_i)\right)}{V_Y}$$

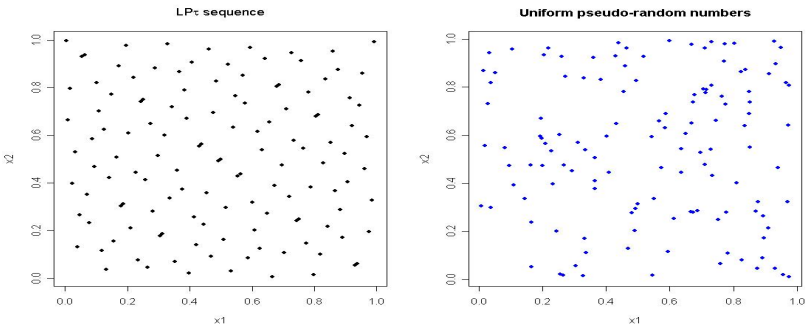
Total-order Sensitivity Measure (TS_i)

The sum of all the sensitivity measures involving the factor in question. e.g. for a model with three input factors, $TS_1 = S_1 + S_{12} + S_{13} + S_{123}$.

$$TS_i \equiv \frac{E_{\mathbf{x}_{-i}}\left(V_{x_i}(Y|\mathbf{x}_{-i})\right)}{V_Y}$$

Sobol' LP$_\tau$ sampling

- Each Sensitivity Measure is a quotient of integrals in a multidimensional space, which can be approximated via MC integration.
- For large or computer-intensive models it is important that the integral be approximated with as few model evaluations as possible.
- The LP$_\tau$ sequences have the property of always generating points which are regularly distributed in the factor space.



2002 Knowledge Economy Index

	GERD	PhD	RES	TES	GFCF	LLL	e-gov
DE	46502	8651	265812		160154	2692	0.48
FR	31871		186420	5.81	154586	833	0.63
UK	25763	7224	170107		133181		0.62
IT	15013			4.75	149058		0.57
ES	7829	2544	83318	4.44	81584	1121	0.64
BE	5352	711	32856	6.11	27715	358	0.47
AT	4467	843	25328		19490	337	0.56
FI	4176		38632	6.39	9906	533	0.76
DK	3456		25912	8.51	16759	547	0.82
PT	1286	1017		5.83	20191	161	0.58
IE	1167	316			7573	154	0.85
SE		1727		7.66	21870	869	0.87
NL		933		5.08	35402	1476	0.54
EL				3.96	18676		0.52

21 missing values

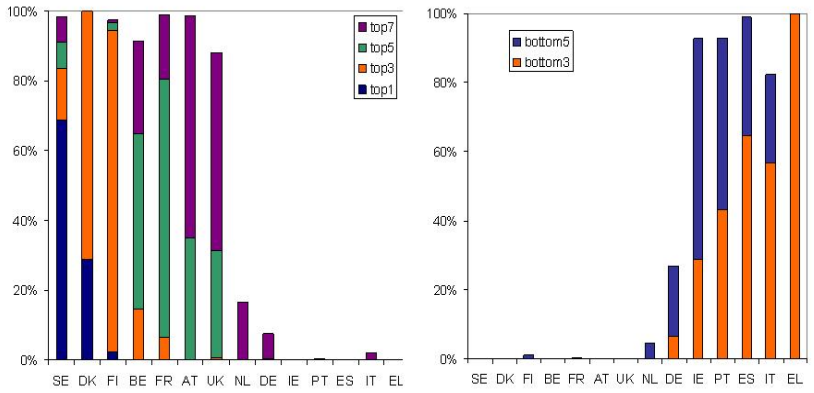
value above the mean

value below the mean

Acknowledging assumptions in the development of the Index

1. **Selecting Indicators**
   Inclusion- Exclusion of one indicator-at-a-time
2. **Imputation**
   Trend model:
   least squares polynomial regression + t-test for the estimates of the std for regression coefficients
3. **Weighting**
   1. Equal weights
   2. Conceptual model
   3. Country-specific weights
4. **Aggregation**
   1. Linear
   2. Geometric

Uncertainty analysis results



Investing in the Knowledge Economy (EU-15):
AT has a 35% probability to be among the top 5 countries and 0% probability to be among the bottom 5 countries

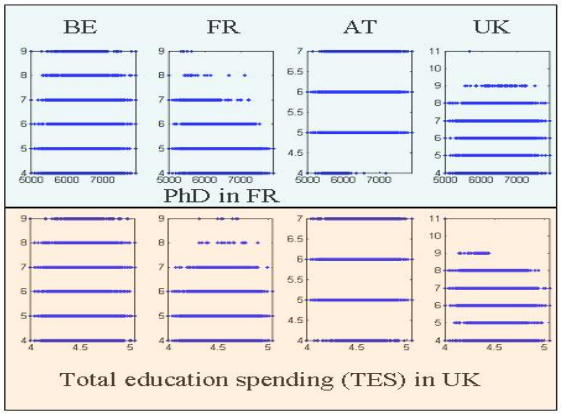
Sensitivity analysis results (Sobol' method)

	BE	BE	FR	FR	AT	AT	UK	UK
	First Order	Total Effect	First Order	Total Effect	First Order	Total Effect	First Order	Total Effect
(13 imputed values)	~0	~0	~0	~0	~0	~0	~0	~0
GERD_EL2002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
PHD_FR2002	0.009	0.045	0.139	0.353	0.049	0.155	0.017	0.022
PHD_FI2002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
RSE_EL2002	0.000	0.001	0.000	0.000	0.000	0.000	0.003	0.003
RSE_SE2002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
TES_UK2002	0.000	0.032	0.000	0.083	0.014	0.072	0.012	0.098
LLL_IT2002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LLL_UK2002	0.004	0.008	0.001	0.024	0.001	0.036	0.002	0.049
Aggregation	0.011	0.139	0.166	0.556	0.251	0.505	0.286	0.462
Weighting	0.052	0.169	0.008	0.207	0.011	0.165	0.079	0.228
Incl./Excl.	0.718	0.894	0.147	0.582	0.351	0.663	0.319	0.531
sum of all 24 input factors	0.804		0.482		0.684		0.718	

First order: Capture individual impact

Total effect: Capture interactions/synergies

Sensitivity analysis as a tool to identify thresholds



Selected countries rank versus two important imputed values:
$PhD_{FR} \sim N(6428,476)$
$TES_{UK} \sim N(4.52,0.17)$

Regardless of the changes in the other factors (imputed values, aggregation, weighting, set of indicators)...

- France will not fall behind the 6^{th} position if the expected number of PhD students is 7200.
- UK will not fall behind the 8^{th} position if the expected value for TES $= 4.52\,\%$ is the correct one.

Further reading

JRC Information Server on Composite Indicators at http://farmweb.jrc.cec.eu.int/ci/

- Nardo M., Saisana M., Saltelli A. and Tarantola S. (2005) Tools for Composite Indicators Building, EUR 21682, European Commission.
- Nardo M., M. Saisana, A. Saltelli and S. Tarantola, A. Hoffman and E. Giovannini (2005) Handbook on Constructing Composite Indicators: Methodology and User Guide OECD Statistics Working Paper JT00188147, STD/DOC(2005)3.http://www.olis.oecd.org/olis/2005doc.nsf/LinkTo/std-doc(2005)3
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