Least Angle Regression

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Overview

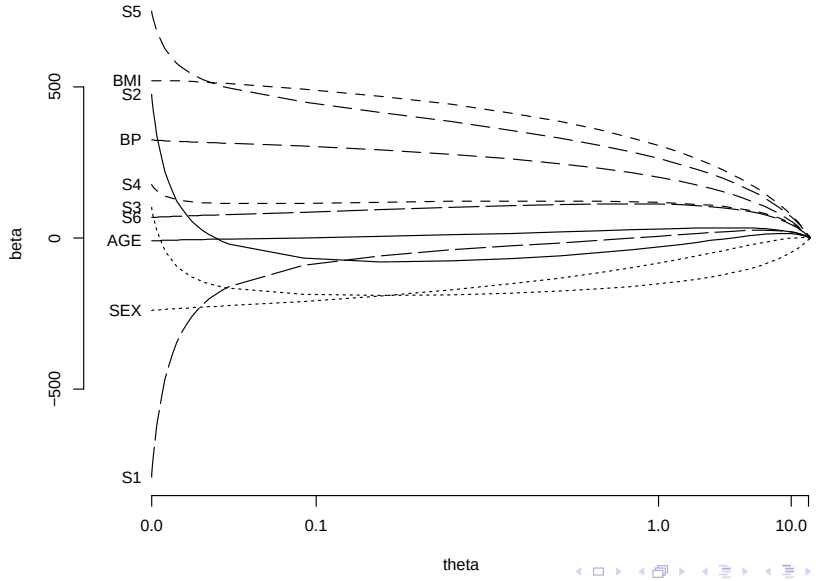
- Why is LARS imporant?
- Other packages
- GLARS package
- Issues
- Insightful Research

Why is LARS important?

- Variable Selection in Regression
  - Important
  - Many approaches: stagewise, boosting, LASSO, regularization, ...
- Least Angle Regression — Efron, Hastie, Johnstone, Tibshirani (2004) Annals (with discussion)
  1. Lasso
  2. Forward stagewise
  3. Least Angle Regression (LAR)
     - Unifying explanation
     - Fast implementation
     - Fast way to choose tuning parameter

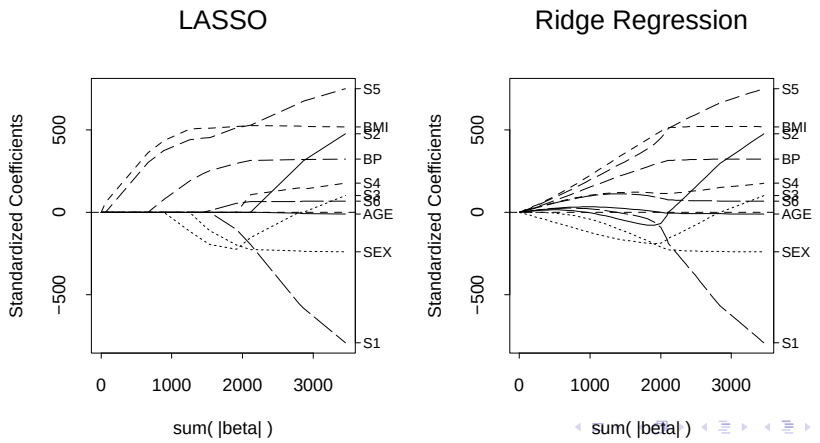
Ridge Regression

- Minimize $\sum(Y_i - \hat{Y}_i) + \lambda \sum \hat{\beta}_j^2$



LASSO

- Minimize $\sum(Y_i - \hat{Y}_i) + \lambda \sum |\hat{\beta}_j|$
- Forces small coefficients $\to 0$; gives simpler models.
- Smaller penalty on large coefficients: less effect on important terms
- Implementation is more complicated and slower

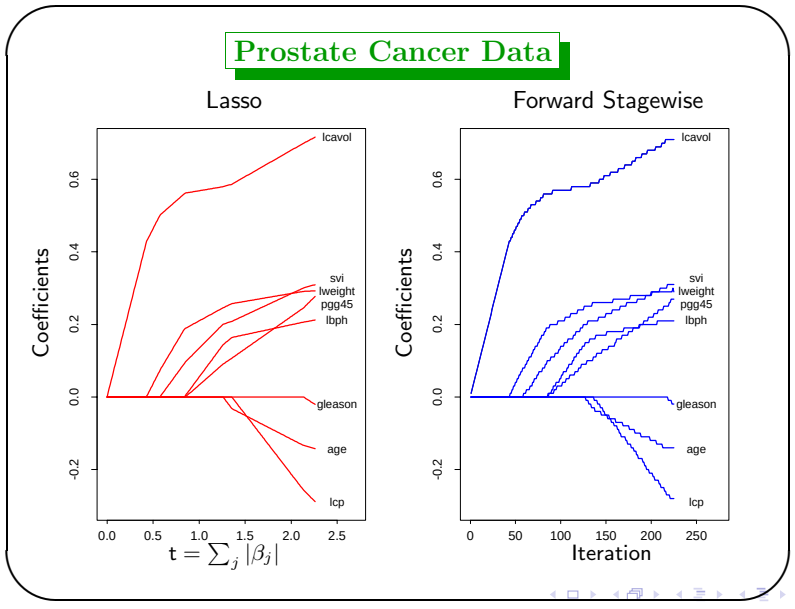


Forward Stagewise Regression

(Forward Stagewise = Least Squares Boosting)

1. Initialize: standardize predictors, center y, $r = y, \beta_1 = \ldots = \beta_p = 0$
2. Repeat many times
   - Find the predictor x_j most correlated with r
   - $\delta = \epsilon \text{sign}(r \cdot x_j)$
   - $\hat{\beta}_j \leftarrow \hat{\beta}_j + \delta$
   - $r \leftarrow r - \delta x_j$

Forward Stagewise and LASSO



Similarity:

Are LASSO and infinitesimal forward stagewise identical?

- With orthogonal predictors, yes.
- Otherwise similar.

Least Angle Regression provides explanation, and fast implementation.

Stepwise, Forward Stagewise, Least Angle

Stepwise regression:

- Pick predictor most correlated with y
- Bring predictor completely into model (full LS fit)

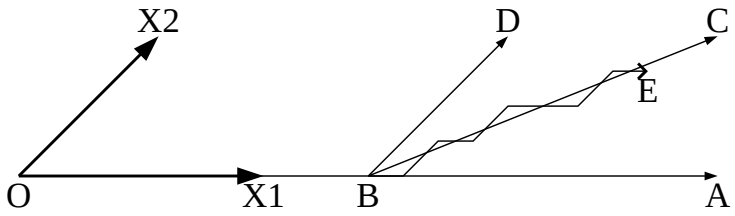
Forward stagewise:

- Pick predictor most correlated with y
- Increment coefficient for predictor

Least Angle Regression:

- Pick predictor most correlated with y
- Bring predictor into model only to extent it is better than others
- Move in least-squares direction until another variable is as correlated

Least Angle Regression



C = projection of y onto space spanned by X_1 and X_2.

B = first step for least-angle regression

E = point on stagewise path

LARS - other packages

lars : Efron and Hastie (S-PLUS and R)

- Linear regression

glmpath : Park and Hastie (R)

- GLM and Cox Proportional Hazards

Methods: `plot`, `print`, `predict`, `cv`, `coef`

S+GLARS

- S-PLUS and R, open source
  - Incorporate `lars`, `glmpath`
  - Cleanup, consistent interface
  - Incorporate future work by others; provide framework
- Extensions
  - Numerically-accurate calculations
  - Factors, splines, polynomials, interactions, ...
  - Other models (robust regression, ...), other penalties
  - Missing data
  - Massive data sets
  - Diagnostics, tools for selecting tuning parameter
- User-friendly
  - Consistent interface
  - GUI
  - Documentation

Issues

- Money
  - NIH funding: require commercial potential
  - Insightful: indirect benefit
- Outside contributors
- Licensing; ability to ship with S-PLUS, I-Miner.

Insightful Research Department

- Turn research into software for wide use
  - Higher standards than academic software (ease of use, robustness, testing)
- Collaboration
- Variety: resampling, missing data, group sequential designs, simulation-based econometric software, functional data, stable distributions, proteomics, microarrays, frailty models, causal modeling
- External funding — SBIR grants (NIH, NSF, . . .)
  - Somewhat easier funding
  - Commercial potential
  - Risk, research element
- We're hiring
- We're looking for good projects and collaborators