Iterated function system and simulation of Brownian motion

Stefano Iacus and Davide La Torre

Several methods are currently available to simulate paths of the Brownian motion. In particular, paths of the BM can be simulated using the properties of the increments of the process like in the Euler scheme, or as the limit of a random walk or via L2 decomposition like the Kac-Siegert/Karnounen-Loeve series.

In Iacus and La Torre (2006, see http://arxiv.org/abs/math.PR/0601379) a IFSM (Iterated Function Systems with Maps) operator whose fixed point is the trajectory of the BM is proposed. In that paper we studied the application of this representation of stochastic processes to simulate their trajectories. The resulting simulated trajectories are self-affine, continuous and fractal by construction.

This fact produces more realistic trajectories than other schemes in the sense that their geometry is closer to the one of the true BM's trajectories. The IFSM trajectory of the BM can then be used to generate more realistic solutions of stochastic differential equations.

Pathwise approximations of stochastic processes remain a relevant topic, for example, in computational finance and numerical option pricing. In this work we will discuss some advances on this topic and present the new version of the IFS package.