Structured Bradley-Terry models, and flat lizards fighting

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Abstract

The Bradley-Terry model (Bradley and Terry, 1952; Agresti, 2002) is a useful device for the scoring of 'tournaments' for which the data are the results of 'contests' between pairs of 'players'. Applications are many, ranging from bibliometrics (Stigler, 1994) in which the 'players' are academic journals, to genetics (for example, the allelic transmission/disequilibrium test of Sham and Curtis (1995) is based on a Bradley-Terry model in which the 'players' are alleles). The Bradley-Terry model in its simplest form, with no ties permitted, is a logistic regression with a specially structured design matrix: in any contest between players i and j,

$$\text{logit}[\text{pr}(i \text{ beats } j)] = \lambda_i - \lambda_j.$$

A simple elaboration allows an 'order' effect, for example to allow one player in each contest to enjoy an advantage on account of 'going first', or 'playing on home turf':

$$\text{logit}[\text{pr}(i \text{ beats } j \text{ in contest } t)] = \lambda_i - \lambda_j + \delta z_t,$$

where $z_t = 1$ if i has the supposed advantage and $z_t = -1$ if j has it. (If the 'advantage' is in fact a disadvantage, δ will be negative.) The scores λ_i then relate to ability in the absence of any such advantage.

Motivated by study of a large 'tournament' among male flat lizards (*Platysaurus broadleyi*) in the wild, we consider structured forms of the Bradley-Terry model in which the ability parameters $\lambda_1, \ldots, \lambda_K$ are determined by measured attributes of the K players involved, *viz.*

$$\lambda_i = \sum_{r=1}^{p} \beta_r x_{ir} \quad (i = 1, \ldots, K).$$

Special attention is needed in the case of contests involving any lizard i whose explanatory values $x_{i1}, \ldots, x_{ip}$ are incomplete.

This talk will describe the *BradleyTerry* package for *R* (Firth, 2004), which provides facilities for the specification and fitting of such models, for model selection by standard methods, and for model criticism *via* special-purpose residuals. Use of the package will be illustrated using the lizard data ($K = 77$), analysis of which reveals biologically-important evidence on the role played by bright colours on a male lizard's body.

(Arising from joint work with M J Whiting, D M Stuart-Fox, D O'Connor, N Bennett and S Blomberg.)

References

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