Generalized Significance in Scale Space: The GS3 Package

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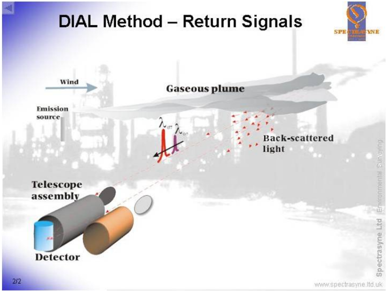

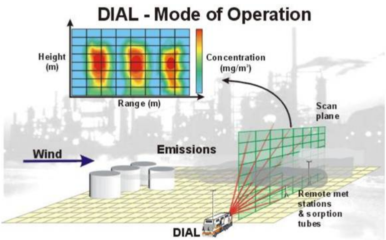

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- **H**igh **S**pectral **R**esolution **L**IDAR
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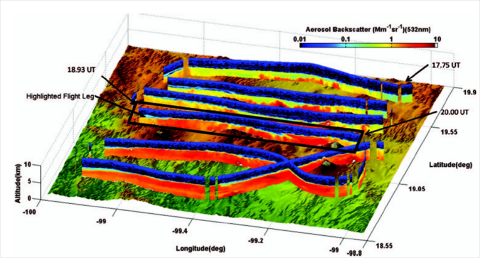

Hair et al. (2008)

Data graciously provided by NASA Langley Research Center

Challenges associated w/ HSRL & DIAL data

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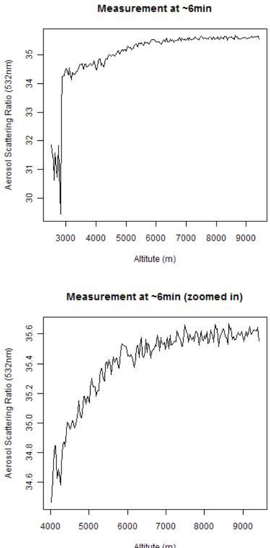

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- Highly variable
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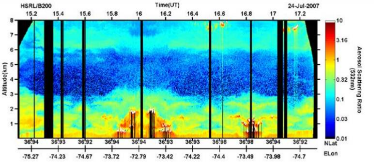

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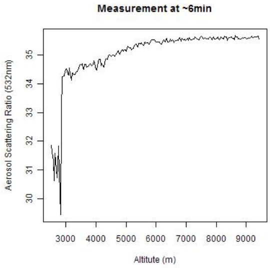

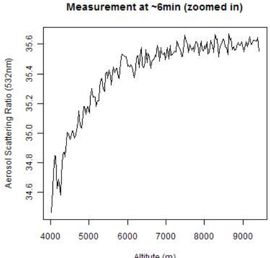

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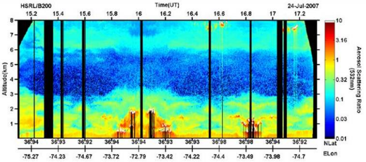

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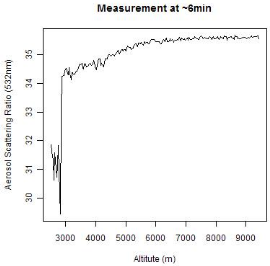
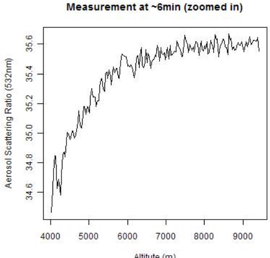
- Large ($\sim 300 \times 30,000$)





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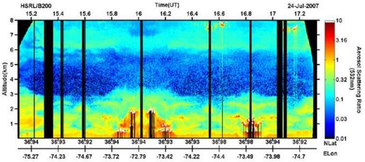

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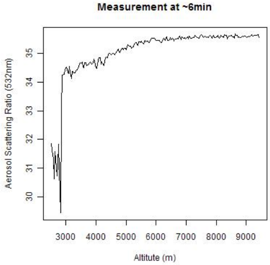
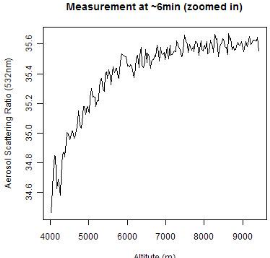
Goals

- Estimate concentration (derivative)
- Calculate uncertainty





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The GS3 package provides *a* solution

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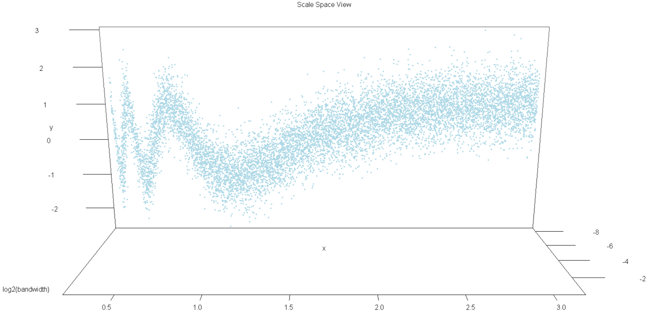
Scale space (Chaudhuri & Marron (2000)) a good starting point.
Consider the following example:



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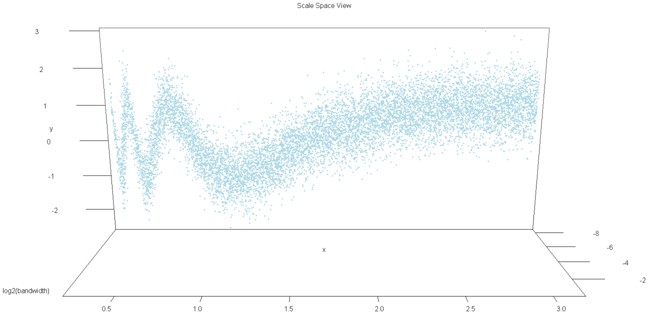

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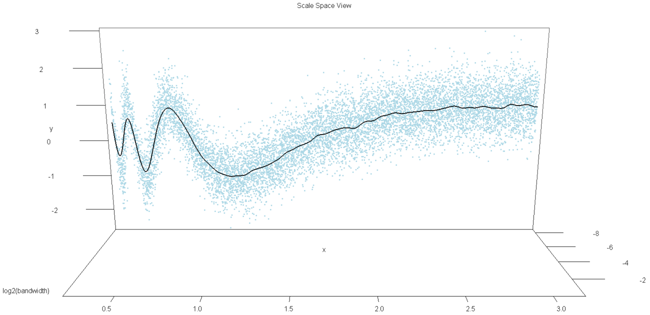

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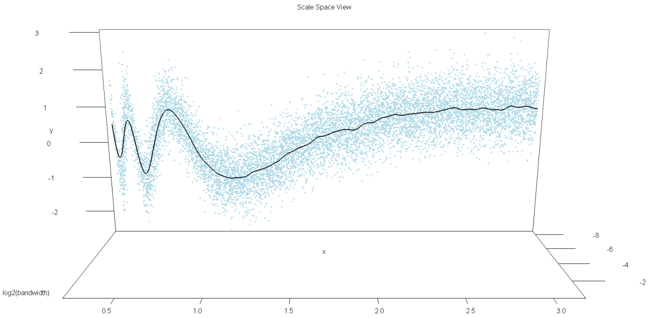

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Consider the following example:

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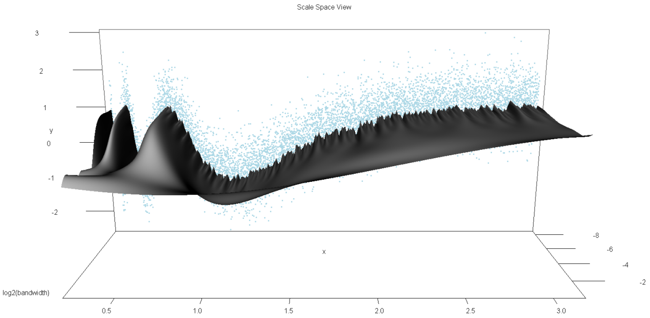
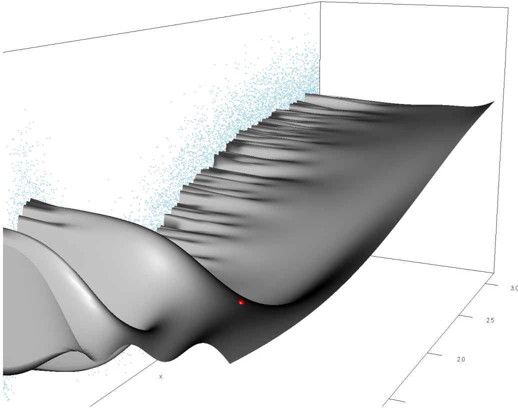
- Scale space studies a "family" of smooths



RODEO

RODEO (Wasserman & Lafferty (2008)) greedy algorithm for traversing "scale space surface"

Algorithm



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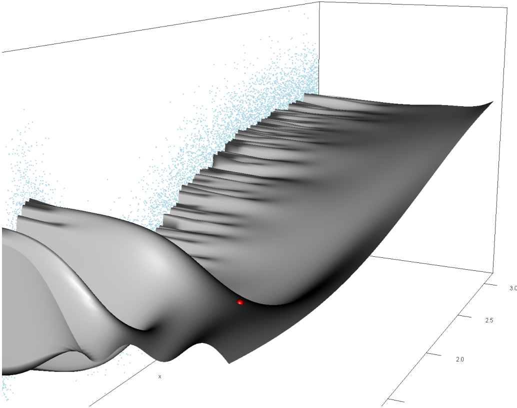

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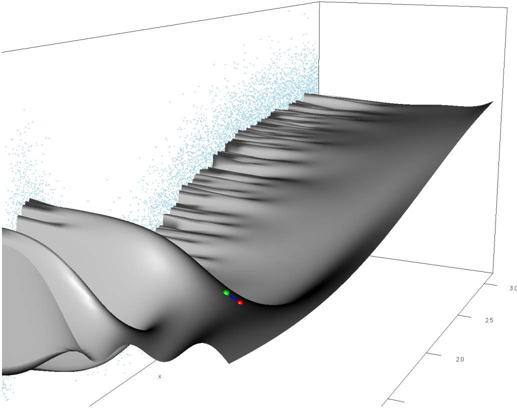

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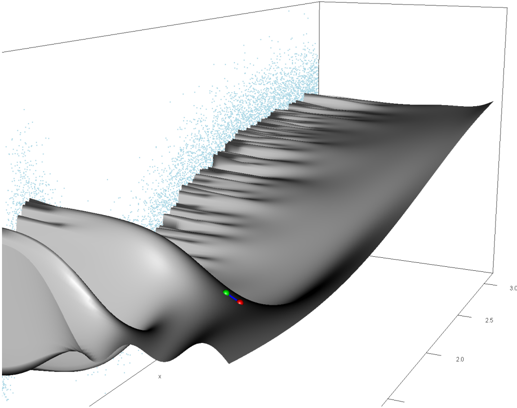

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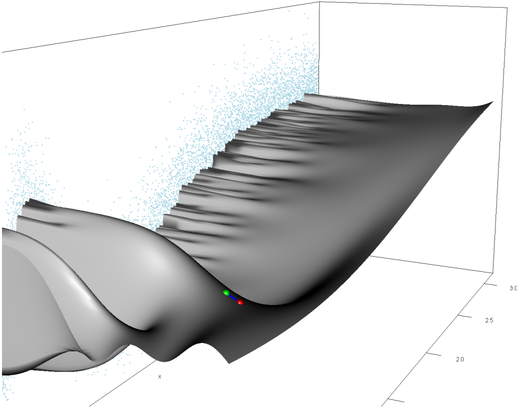

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- NB: $\mathrm{Var}(Z) \sim \sigma^2$
  - σ^2 unknown population parameter

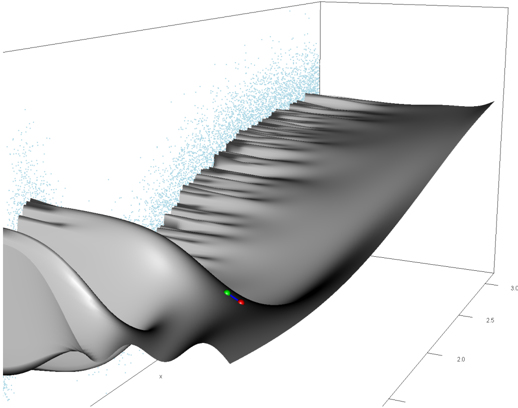
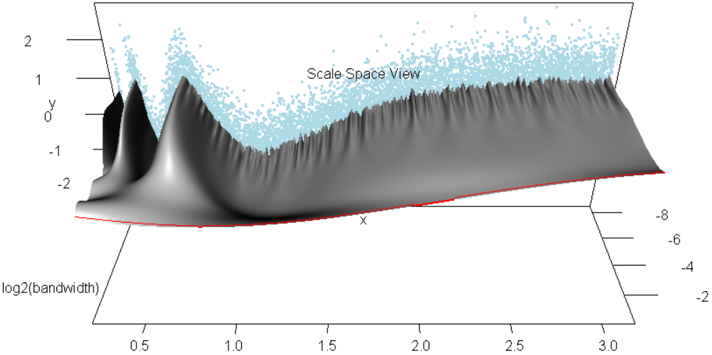


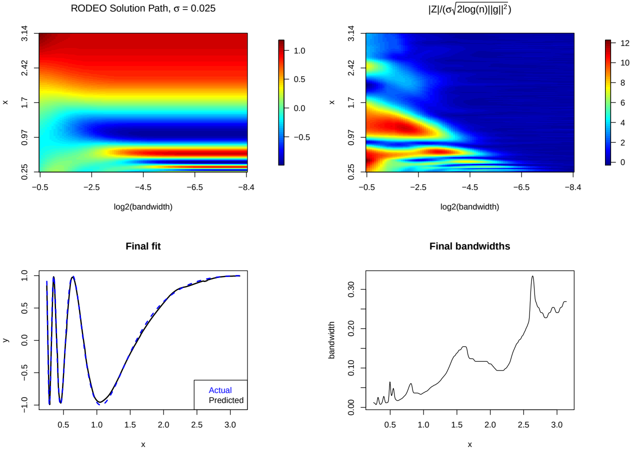
Illustration of RODEO in Scale Space

For $\sigma = 0.025$



Results

- Produces sensible smooth & bandwidths



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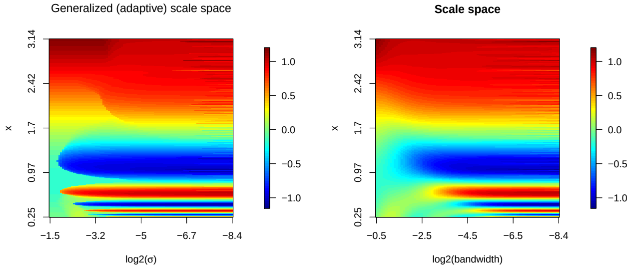
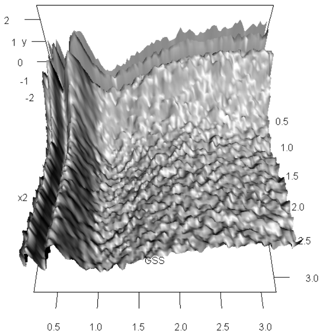


Higher dimensions

$d > 1$

- Straightforward, calculate Z_j, $j = 1, \ldots, d$ & $\mathrm{Var}(Z_j)$ $(\sim \sigma)$
- Same greedy approach used along each dimension

Generalized Scale Space



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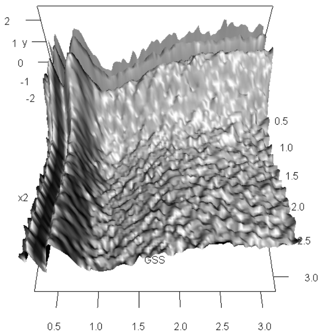
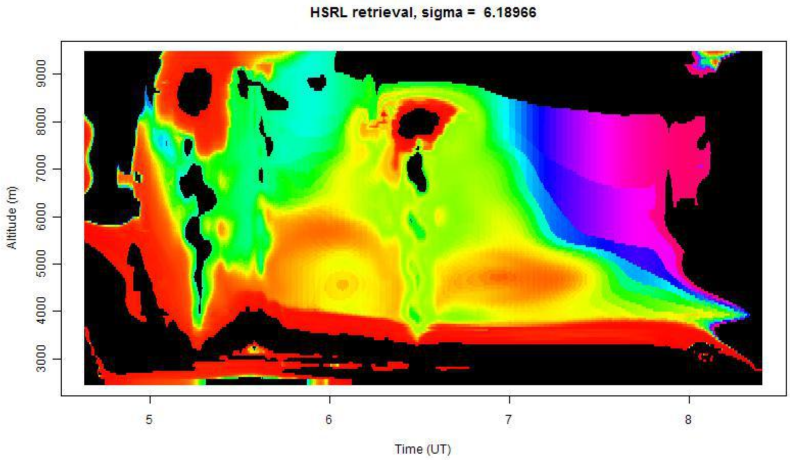


Computation

Speed of Bandwidth Selection

- Use linear binning ideas (Fan & Marron (1994), Wand (1995))
- M_i, $i = 1, \ldots, d$ grid size & $T = \#$ candidate bandwidths
- Selection takes $O(dTM_1 \log M_1 \ldots M_d \log M_d)$
- Compared to smooth w/ global bandwidth, $O(M_1 \log M_1 \ldots M_d \log M_d)$

Illustration of Generalized Scale Space on HSRL data



Conclusions & Future Work

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Future Work (currently in progress)

- Local variance function estimation
- Spatial dependence