Exploratory Analysis of a Large Collection of Time-Series Using Automatic Smoothing Techniques

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- Goal: To extract summary measures and features from a large collection of time series.
  1. Exploratory analysis (as opposed to inferential)
  2. Hypothesis generation
  3. Interesting (anomalous) time series
  4. Common features among time series (e.g., critical points)
- Process to be as automatic as possible.

What do we mean by features?

- Scale of time series
- Mean value of function
- Values of derivatives
- Outliers
- Critical points
- Curvatures
- Signal/noise
- Others

- Features are defined on smooth curves.
- What we have is discretely sampled observations.
- We need functional data techniques to recover underlying smooth function.

$$y(t_i) = f(t_i) + \varepsilon_i; \;\; E(\varepsilon_i) = 0$$

- Automatic bandwidth selection procedures (e.g., cross-validation, plug-in)

- Optimal bandwidth selection is usually applied to the function.
- This may NOT be optimal for estimating derivatives.
- The relationship between optimal BWs for function estimation and derivative estimation is not clear.
- Here we evaluate 4 automatic smoothing techniques in terms of their accuracy for estimating functions and its first two derivatives via simulation studies.

- Smoothing splines with gcv for bw selection (*stats::smooth.spline*).
- Penalized splines with REML estimate(*SemiPar::spm*).
- Local polynomial with plugin bw (*KernSmooth::locpoly*).
- Gasser-Muller kernel global plug-in bw (*lokern::glkerns*).

- Regression function. (4 functions with different characteristics)
- Error distribution. (t distribution 5 df)
- Grid layout. (either uniform random or equally spaced)
- Noise level. ($\sigma = 0.5$, 1.2)

Regression Function Estimation

MISE, Variance & Bias^2

Function	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2), \quad \sigma = 0.5,$	2.60 2.600 0.031	0.36 0.100 0.250	0.16 0.100 0.057	0.18 0.069 0.110
$f_2(x) = [1 + \exp(-10x)]^{-1}, \quad \sigma = 0.5,$	2.100 2.100 0.0041	0.026 0.026 0.0000	0.049 0.048 0.0000	0.028 0.028 0.0000
$f_3(x) = 10\exp(-x/60) + 0.5\sin(\frac{2\pi}{20}(x-10)) + \sin(\frac{2\pi}{20}(x-30))$ $\sigma = 0.5$	0.00540 0.00540 $5.4e-05$	0.02200 0.00020 0.021	0.00081 0.00068 0.00013	0.00084 0.00060 0.00025
$f_4(x) = \sin(8\pi x^2), \quad \sigma = 0.5,$	0.048 0.043 0.0091	0.640 0.120 0.5200	0.068 0.042 0.0270	0.089 0.027 0.0620

First Derivative Estimation

MISE, Variance & Bias^2

First Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2)$, $\sigma = 0.5$,	44.00 44.00 0.21	0.80 0.11 0.69	0.47 0.16 0.30	0.66 0.28 0.38
$f_2(x) = [1 + \exp(-10x)]^{-1}$, $\sigma = 0.5$,	2600.00 2600.00 6.300	0.67 0.57 0.098	3.20 3.20 0.014	2.90 2.90 0.018
$f_3(x) = 10\exp(-x/60) + 0.5\sin(\frac{2\pi}{20}(x-10)) + \sin(\frac{2\pi}{20}(x-30))$ $\sigma = 0.5$	25.000 25.000 0.047	0.970 0.0023 0.970	0.055 0.0400 0.015	0.090 0.0820 0.008
$f_4(x) = \sin(8\pi x^2)$, $\sigma = 0.5$,	0.13 0.098 0.037	0.73 0.130 0.610	0.17 0.041 0.130	0.15 0.047 0.110

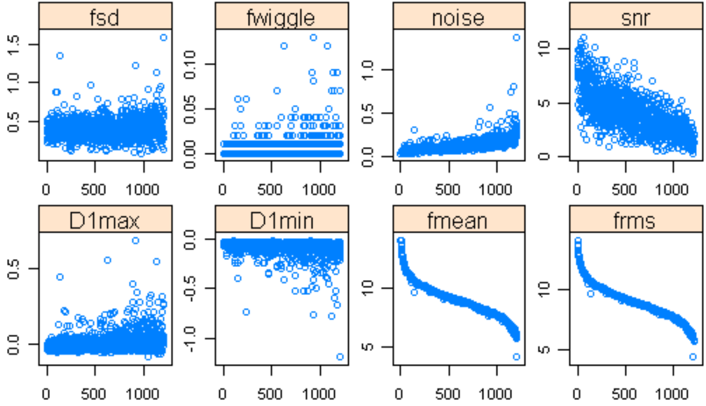
Second Derivative Estimation

MISE, Variance & Bias^2

Second Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2), \quad \sigma = 0.5,$	230.00 230.00 1.00	1.00 0.001 1.00	0.99 0.015 0.97	1.00 0.079 0.96
$f_2(x) = [1 + \exp(-10x)]^{-1}, \quad \sigma = 0.5,$	$6.6e+06$ $6.6e+06$ 14000.0	6.90 3.40 3.50	217.0 214.0 3.00	482.0 478.0 3.6
$f_3(x) = 10\exp(-x/60) + 0.5\sin(\frac{2\pi}{20}(x-10)) + \sin(\frac{2\pi}{20}(x-30))$ $\sigma = 0.5$	4600.00 4.6e03 7.800	1.00 0.0015 1.000	0.23 0.11 0.120	2.50 2.50 0.019
$f_4(x) = \sin(8\pi x^2), \quad \sigma = 0.5,$	0.81 0.730 0.084	0.80 0.160 0.640	0.32 0.035 0.290	0.41 0.280 0.130

- Smoothing spline, with cross-validated optimal bandwidth, did poorly.
- Penalized splines, with REML penalty estimation, did well on smooth functions, and worse on functions with high frequency variations (high bias).
- Global plug-in bandwidth kernel methods, *glkerns* and *locpoly* generally did well (higher variance).
- *glkerns* seems to be a good choice for estimating lower-order derivatives.

- An R function to extract summary measures and features of a collection of time series.
- We demonstrate that with a large collection of time series data from AT&T.
- Over 1200 time-series with monthly MOU over a 3.5 year period.
- The data were transformed & scaled for proprietary reasons.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fwiggle
noise
snr
D1max
D1min
fmean
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500
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A Biplot on Features

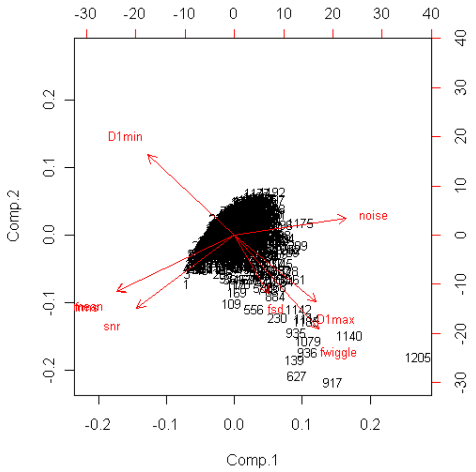

Figure: PCA of features Data

ts: 1205 ts: 1140

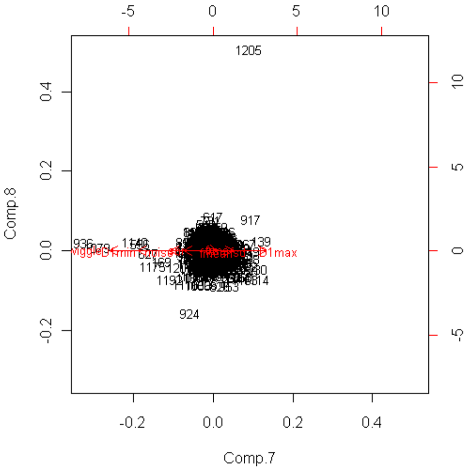

Figure: PCA of features Data

ts: 139 ts: 936 Next

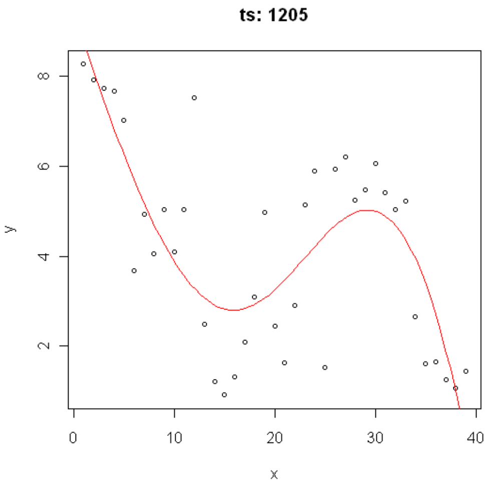

FIGURE: PCA of features Data

Back to PCA

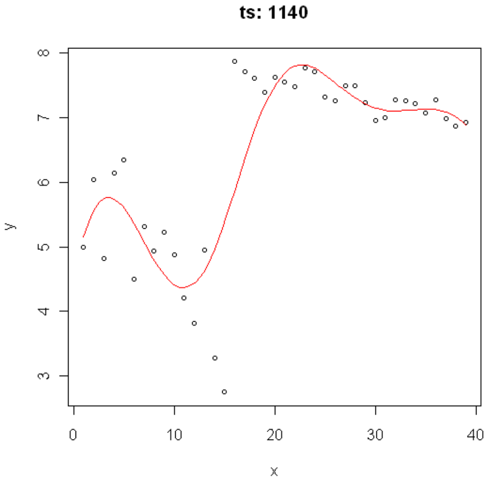

FIGURE: PCA of features Data

Back to PCA

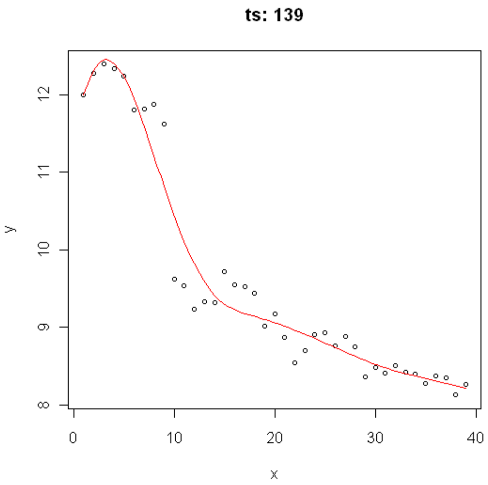

FIGURE: PCA of features Data

Back to PCA

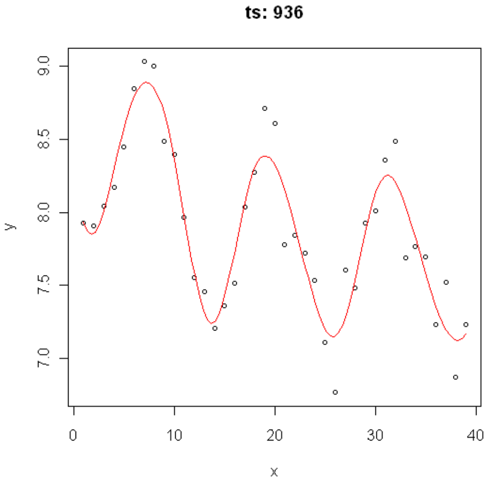

FIGURE: PCA of features Data

Back to PCA

Future Work

- Release package.
- Add more visualization.
- Further testing on real data.

THANK YOU!

Semiparametric Model Details

- Nonparametric regression models are used.

Functional form of the models

- We consider a univariate scatterplot smoothing $y_i = f(x_i) + \epsilon_i$ where the (x_i, y_i), $1 \leq i \leq n$, are scatter plot data, ϵ_i are zero mean random variables with variance σ_ϵ^2 and $f(x) = E(y|x)$ is a smooth function.
- f is estimated using penalised spline smoothing using truncated polynomial basis functions. These involve f being modelled as a function of the form

$$f(x) = \beta_0 + \beta_1 x + \cdots + \beta_p x^p + \sum_{k=1}^{K} u_k (x - x_k)^p$$

where u_k are random coefficients

$$u \equiv [u_1, u_2, \ldots, u_K]^T \sim N(0, \sigma_u^2 \, \Omega^{-1/2} \, (\Omega^{-1/2})^T), \quad \Omega \equiv [|x_k - x_{k'}|^{2p}]$$

- The mixed model representation of penalised spline smoothers allows for automatic fitting using the R linear mixed model function. Smoothing parameter selection is done using REML and $\hat{f}(x)$ is obtained via best linear unbiased prediction.
- This class of penalised spline smoothers may also be expressed as

$$\hat{f} = C(C^T C + \lambda^{2p} D)^{-1} \, C^T y$$

where $\lambda = \frac{\sigma_u^2}{\sigma_\epsilon^2}$ is the smoothing parameter,

$$C \equiv [1, x_i, \ldots, x_i^{m-1} |x_i - x_k|^{2p}]$$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

Function	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2), \quad \sigma = 0.5,$	$(2.100)_{MISE} = (2.100)_{ivar} + (0.0041)_{isb}$	$(0.026)_{MISE} = (0.026)_{ivar} + (0.0000)_{isb}$	$(0.049)_{MISE} = (0.048)_{ivar} + (0.0000)_{isb}$	$(0.028)_{MISE} = (0.028)_{ivar} + (0.0000)_{isb}$
$f_2(x) = [1 + \exp -10x]^{-1}, \quad \sigma = 0.5,$	$(1.30)_{MISE} = (1.30)_{ivar} + (0.10)_{isb}$	$(0.68)_{MISE} = (0.21)_{ivar} + (0.470)_{isb}$	$(0.31)_{MISE} = (0.25)_{ivar} + (0.065)_{isb}$	$(0.27)_{MISE} = (0.22)_{ivar} + (0.055)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.4,$	$(2.30)_{MISE} = (2.20)_{ivar} + (0.059)_{isb}$	$(0.48)_{MISE} = (0.23)_{ivar} + (0.260)_{isb}$	$(0.43)_{MISE} = (0.34)_{ivar} + (0.093)_{isb}$	$(0.36)_{MISE} = (0.30)_{ivar} + (0.060)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 4,$	$(9.40)_{MISE} = (9.40)_{ivar} + (0.0430)_{isb}$	$(0.63)_{MISE} = (0.63)_{ivar} + (0.0000)_{isb}$	$(0.95)_{MISE} = (0.95)_{ivar} + (0.0093)_{isb}$	$(0.89)_{MISE} = (0.89)_{ivar} + (0.0078)_{isb}$
$f_5(x) = a\exp(-bx) + k_1\sin(\frac{2\pi}{T_1}(x-10) + k_2\sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(0.00540)_{MISE} = (0.00540)_{ivar} + (5.4e-05)_{isb}$	$(0.02200)_{MISE} = (0.00020)_{ivar} + (2.1e-02)_{isb}$	$(0.00081)_{MISE} = (0.00068)_{ivar} + (1.3e-04)_{isb}$	$(0.00084)_{MISE} = (0.00060)_{ivar} + (2.5e-04)_{isb}$
$f_6(x) = \sin(8\pi x^2), \quad \sigma = 0.5,$	$(0.048)_{MISE} = (0.043)_{ivar} + (0.0091)_{isb}$	$(0.640)_{MISE} = (0.120)_{ivar} + (0.5200)_{isb}$	$(0.068)_{MISE} = (0.042)_{ivar} + (0.0270)_{isb}$	$(0.089)_{MISE} = (0.027)_{ivar} + (0.0620)_{isb}$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

First Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2), \quad \sigma = 1.6,$	$(44.00)_{MISE} = (44.00)_{ivar} + (0.21)_{isb}$	$(0.80)_{MISE} = (0.11)_{ivar} + (0.69)_{isb}$	$(0.47)_{MISE} = (0.16)_{ivar} + (0.30)_{isb}$	$(0.66)_{MISE} = (0.28)_{ivar} + (0.38)_{isb}$
$f_2(x) = [1 + \exp -10x]^{-1}, \quad \sigma = 1.2,$	$(2600.00)_{MISE} = (2600.00)_{ivar} + (6.300)_{isb}$	$(0.67)_{MISE} = (0.57)_{ivar} + (0.098)_{isb}$	$(3.20)_{MISE} = (3.20)_{ivar} + (0.014)_{isb}$	$(2.90)_{MISE} = (2.90)_{ivar} + (0.018)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.4,$	$(490.00)_{MISE} = (490.00)_{ivar} + (0.52)_{isb}$	$(1.00)_{MISE} = (0.26)_{ivar} + (0.75)_{isb}$	$(0.95)_{MISE} = (0.62)_{ivar} + (0.33)_{isb}$	$(1.50)_{MISE} = (1.20)_{ivar} + (0.23)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 4,$	$(33000.0)_{MISE} = (33000.0)_{ivar} + (20.000)_{isb}$	$(5.3)_{MISE} = (5.2)_{ivar} + (0.086)_{isb}$	$(26.0)_{MISE} = (26.0)_{ivar} + (0.033)_{isb}$	$(40.0)_{MISE} = (40.0)_{ivar} + (0.048)_{isb}$
$f_5(x) = a\exp(-bx) + k_1\sin(\frac{2\pi}{T_1}(x-10) + k_2\sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(25.000)_{MISE} = (25.0000)_{ivar} + (0.047)_{isb}$	$(0.970)_{MISE} = (0.0023)_{ivar} + (0.970)_{isb}$	$(0.055)_{MISE} = (0.0400)_{ivar} + (0.015)_{isb}$	$(0.090)_{MISE} = (0.0820)_{ivar} + (0.008)_{isb}$
$f_6(x) = \sin(8\pi x^2), \quad \sigma = 0.5,$	$(0.13)_{MISE} = (0.098)_{ivar} + (0.037)_{isb}$	$(0.73)_{MISE} = (0.130)_{ivar} + (0.610)_{isb}$	$(0.17)_{MISE} = (0.041)_{ivar} + (0.130)_{isb}$	$(0.15)_{MISE} = (0.047)_{ivar} + (0.110)_{isb}$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

Second Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-400x^2), \quad \sigma = 1.6,$	$(230.00)_{MISE} = (230.00)_{ivar} + (1.00)_{isb}$	$(1.00)_{MISE} = (0.001)_{ivar} + (1.00)_{isb}$	$(0.99)_{MISE} = (0.015)_{ivar} + (0.97)_{isb}$	$(1.00)_{MISE} = (0.079)_{ivar} + (0.96)_{isb}$
$f_2(x) = [1 + \exp -10x]^{-1}, \quad \sigma = 1.2,$	$(6.6e + 06)_{MISE} = (6.6e + 06)_{ivar} + (14000.0)_{isb}$	$(6.9e + 00)_{MISE} = (3.4e{+}00)_{ivar} + (3.5)_{isb}$	$(2.2e + 02)_{MISE} = (2.1e{+}02)_{ivar} + (3.0)_{isb}$	$(4.8e + 02)_{MISE} = (4.8e{+}02)_{ivar} + (3.6)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.4,$	$(1.4e + 05)_{MISE} = (1.4e + 05)_{ivar} + (95.00)_{isb}$	$(1.1e + 00)_{MISE} = (1.5e - 01)_{ivar} + (0.94)_{isb}$	$(1.8e + 00)_{MISE} = (1.2e + 00)_{ivar} + (0.62)_{isb}$	$(3.7e + 01)_{MISE} = (3.7e + 01)_{ivar} + (0.44)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 4,$	$(3.7e + 10)_{MISE} = (3.7e{+}10)_{ivar} + (1.4e{+}07)_{isb}$	$(6.5e + 01)_{MISE} = (6.5e{+}01)_{ivar} + (6.6e{-}01)_{isb}$	$(1.0e + 03)_{MISE} = (1.0e{+}03)_{ivar} + (1.0e{+}00)_{isb}$	$(3.4e + 04)_{MISE} = (3.4e{+}04)_{ivar} + (3.2e{+}01)_{isb}$
$f_5(x) = a\exp(-bx) + k_1\sin(\frac{2\pi}{T_1}(x-10) + k_2\sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(4600.00)_{MISE} = (4.6e + 03)_{ivar} + (7.800)_{isb}$	$(1.00)_{MISE} = (1.5e - 03)_{ivar} + (1.000)_{isb}$	$(0.231.)_{MISE} = (1e - 01)_{ivar} + (0.120)_{isb}$	$(2.50)_{MISE} = (2.5e + 00)_{ivar} + (0.019)_{isb}$
$f_6(x) = \sin(8\pi x^2), \quad \sigma = 0.5,$	$(0.81)_{MISE} = (0.730)_{ivar} + (0.084)_{isb}$	$(0.80)_{MISE} = (0.160)_{ivar} + (0.640)_{isb}$	$(0.32)_{MISE} = (0.035)_{ivar} + (0.290)_{isb}$	$(0.41)_{MISE} = (0.280)_{ivar} + (0.130)_{isb}$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

Function	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-16x^2), \quad \sigma = 0.4,$	$(0.083)_{MISE} = (0.080)_{ivar} + (0.0029)_{isb}$	$(0.031)_{MISE} = (0.015)_{ivar} + (0.0160)_{isb}$	$(0.022)_{MISE} = (0.017)_{ivar} + (0.0043)_{isb}$	$(0.021)_{MISE} = (0.014)_{ivar} + (0.0071)_{isb}$
$f_2(x) = \sin(2\pi x) + 2\exp(-16x^2), \quad \sigma = 0.3,$	$(0.092)_{MISE} = (0.089)_{ivar} + (0.0034)_{isb}$	$(0.079)_{MISE} = (0.046)_{ivar} + (0.0320)_{isb}$	$(0.035)_{MISE} = (0.026)_{ivar} + (0.0091)_{isb}$	$(0.033)_{MISE} = (0.023)_{ivar} + (0.100)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.1,$	$(0.160)_{MISE} = (0.150)_{ivar} + (0.000)_{isb}$	$(0.055)_{MISE} = (0.049)_{ivar} + (0.012)_{isb}$	$(0.051)_{MISE} = (0.050)_{ivar} + (0.000)_{isb}$	$(0.050)_{MISE} = (0.050)_{ivar} + (0.001)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 1,$	$(0.600)_{MISE} = (0.600)_{ivar} + (0.00180)_{isb}$	$(0.041)_{MISE} = (0.039)_{ivar} + (0.00000)_{isb}$	$(0.078)_{MISE} = (0.073)_{ivar} + (0.00055)_{isb}$	$(0.064)_{MISE} = (0.060)_{ivar} + (0.00018)_{isb}$
$f_5(x) = a\exp(-bx) + k_1 \sin(\frac{2\pi}{T_1}(x-10) + k_2 \sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(2.020293e-07)_{MISE} = (1.930502e-07)_{ivar} + (1.032594e-08)_{isb}$	$(1.526443e-07)_{MISE} = (1.481548e-079)_{ivar} + (6.734309e-0)_{isb}$	$(2.379456e-07)_{MISE} = (2.020293e-07)_{ivar} + (3.456945e-08)_{isb}$	$(2.469247e-07)_{MISE} = (1.795816e-07)_{ivar} + (6.734309e-08)_{isb}$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

First Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-16x^2), \quad \sigma = 0.4,$	$(55.00)_{MISE} = (55.00)_{ivar} + (0.086)_{isb}$	$(0.36)_{MISE} = (0.12)_{ivar} + (0.240)_{isb}$	$(0.27)_{MISE} = (0.15)_{ivar} + (0.120)_{isb}$	$(0.38)_{MISE} = (0.28)_{ivar} + (0.099)_{isb}$
$f_2(x) = \sin(2\pi x) + 2\exp(-16x^2), \quad \sigma = 0.3,$	$(8.0)_{MISE} = (8.000)_{ivar} + (0.013)_{isb}$	$(0.13)_{MISE} = (0.071)_{ivar} + (0.055)_{isb}$	$(0.08)_{MISE} = (0.048)_{ivar} + (0.032)_{isb}$	$(0.19)_{MISE} = (0.170)_{ivar} + (0.013)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.1,$	$(31.00)_{MISE} = (31.00)_{ivar} + (0.049)_{isb}$	$(0.24)_{MISE} = (0.10)_{ivar} + (0.140)_{isb}$	$(0.20)_{MISE} = (0.12)_{ivar} + (0.084)_{isb}$	$(0.32)_{MISE} = (0.27)_{ivar} + (0.048)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 1,$	$(2100.00)_{MISE} = (2100.00)_{ivar} + (1.3000)_{isb}$	$(0.41)_{MISE} = (0.34)_{ivar} + (0.0750)_{isb}$	$(1.80)_{MISE} = (1.80)_{ivar} + (0.0087)_{isb}$	$(2.80)_{MISE} = (2.80)_{ivar} + (0.0077)_{isb}$
$f_5(x) = a\exp(-bx) + k_1\sin(\frac{2\pi}{T_1}(x-10) + k_2\sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(0.25882353)_{MISE} = (0.25882353)_{ivar} + (0.0001176471)_{isb}$	$(0.01411765)_{MISE} = (0.00917647)_{ivar} + (0.0057647059)_{isb}$	$(0.03176471)_{MISE} = (0.02235294)_{ivar} + (0.0092941176)_{isb}$	$(0.30588235)_{MISE} = (0.30588235)_{ivar} + (0.0006705882)_{isb}$

Simulation Output:

Integrated Mean Sq. error, Variance & Bias (for random interval)

Second Derivative	SS	SPM	GLK	LOC
$f_1(x) = x + 2\exp(-16x^2), \quad \sigma = 0.4,$	$1.8e+04$ $1.8\ (+04)$ 12.00	$8.7e-01$ 1.5×10^{-01} 0.72	$8.8e-01$ $2.4e-01$ 0.63	$.0e+01$ $8.0e+01$ 0.50
$f_2(x) = \sin(2\pi x) + 2\exp(-16x^2), \quad \sigma = 0.3,$	$(2400.00)_{MISE} = (2.4e+03)_{ivar} + (1.60)_{isb}$	$(0.24)_{MISE} = (1.1e-01)_{ivar} + (0.13)_{isb}$	$(0.24)_{MISE} = (8.3e-02)_{ivar} + (0.16)_{isb}$	$(12.00)_{MISE} = (7.9e+02)_{ivar} + (0.88)_{isb}$
$f_3(x) = 0.3\exp(-4(x+1)^2) + 0.7\exp(16(x-1)^2), \quad \sigma = 0.1,$	$(8900.00)_{MISE} = (8900.00)_{ivar} + (6.00)_{isb}$	$(0.52)_{MISE} = (0.16)_{ivar} + (0.35)_{isb}$	$(0.51)_{MISE} = (0.19)_{ivar} + (0.32)_{isb}$	$(15.00)_{MISE} = (14.00)_{ivar} + (0.12)_{isb}$
$f_4(x) = 0.8 + \sin(6x), \quad \sigma = 1,$	$(2.3e+09)_{MISE} = (2.3e+09)_{ivar} + (8.7e+05)_{isb}$	$(4.6e+00)_{MISE} = (4.1e+00)_{ivar} + (5.4e-01)_{isb}$	$(6.5e+01)_{MISE} = (6.5e+01)_{ivar} + (1.2e-01)_{isb}$	$(2.1e+03)_{MISE} = (2.1e+03)_{ivar} + (2.1e+00)_{isb}$
$f_5(x) = a\exp(-bx) + k_1\sin(\frac{2\pi}{T_1}(x-10) + k_2\sin(\frac{2\pi}{T_2}(x-30)), \quad \sigma = 0.5,$	$(0.25882353)_{MISE} = (0.25882353)_{ivar} + (0.0001176471)_{isb}$	$(0.01411765)_{MISE} = (0.00917647)_{ivar} + (0.0057647059)_{isb}$	$(0.03176471)_{MISE} = (0.02235294)_{ivar} + (0.0092941176)_{isb}$	$(0.30588235)_{MISE} = (0.30588235)_{ivar} + (0.0006705882)_{isb}$